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## SYMBOLS USED IN MATHEMATICS

S.NO	SYMBOL	DESCRIPTION
1	+	Addition
2	-	Subtraction
3	X	Multiplication
4	÷	Division
5	=	Equal to
6	≠	Not Equal to
7	±	Plus or Minus
8	:	Ratio
9	>	Greater than
10	≥	Greater than or Equal to
11	<	Less than
12	≤	Less than or Equal to
13	< >	Greater than or Less than
14	::	Proportion (is as)
15	≯	Not Greater Than
16	≮	Not Less Than
17	~	Negation
18	Λ	And

19	$\vee$	Or
20	$\infty$	Infinity
21	$\Rightarrow$	Only if
22	$\Leftrightarrow$	If and only if
23	$\exists$	Existential Quantifier (There Exists)
24	$\forall$	Universal Quantifier (for all)
25	$\in$	Belongs to
26	$\notin$	Not Belongs to
27	$\subset$	Subset
28	$\subseteq$	Proper subset
29	$\supseteq$	Super Set
30	/	Such that
31	$\exists$	Such that
32	$\cup$	Union
33	$\cap$	Inter Section
34	$\mu$ or $\cup$	Universal Ser (Mu)
35	$\emptyset$	Empty set or Null set
36	$\bar{A}$ or $A^c$	Complete of A
37	$\Delta$	Symmetry of two sets
38	$P(A)$	Power Set
39	$n(A)$	Cardinal Number
40	( )	Open Interval
41	[ ]	Closed Interval

42	{ }	Flower Brackets or Set Brackets
43		Modulus
44	T	True
45	F	False
46	A	Alpha
47	B	Beta
48	Г	Gama
49	Δ or δ	Delta
50	Ε	Epsilon
51	Λ	Lambda
52	Σ σ or	Sigma
53	W or Ω	Omega
54	Π	Pi
55	≡	Identity
56	≈	Congruence
57	~	Similar to
58	//	Parallel
59	⊥	Perpendicular
60	1°	One degree
61	1ᵍ	One grade
62	1ᶜ	One radian
63	1¹	One minute
64	1¹¹	One second
65	θ	Theta

66	$m$	Slope
67	$\Delta$	Triangle symbol
68	$\square$	Quadrilateral symbol
69	$C$	Constant or intercept
70	$\log$	Logarithm
71	$e$	Exponential
72	$\sqrt{ }$	Square root
73	$\sqrt[3]{ }$	Cube root
74	$\sqrt[n]{ }$	Nth root
75	$\backslash$	Divides
76	$\odot$	Dot circle
77	$\propto$	Proportional
78	$\%$	Percentage
79	$A$	Area (or) amount
80	$P$	Profit or principal
81	$g$	Gain
82	L or (l)	Loss
83	S.I	Simple interest
84	C.I	Compound interest
85	S.P	Selling price
86	C.P	Cost price
87	!	Factorial
88	$L$	Angle
89	$N$	Natural Number Set

90	<b>W</b>	Whole number set
91	<b>Z or I</b>	Set of integers
92	<b>Q</b>	Set of rational numbers
93	<b><math>Q^1</math> or S</b>	Set of irrational numbers
94	<b>R</b>	Set of real numbers
95	<b>I</b>	Set of imaginary numbers
96	<b>C</b>	Set of complex numbers
97	<b>R+</b>	Set of negative real numbers
98	<b>R-</b>	Set of positive real numbers
99	<b>Z+</b>	Positive integers
100	<b>Z-</b>	Negative integers
101	<b>Q+</b>	Set of positive rational numbers
102	<b>Q-</b>	Set of negative rational numbers
103	<b>i</b>	Imaginary number
104	<b>gof</b>	Composite function
105	<b><math>f^{-1}</math></b>	Inverse of F
106	<b><math>A \xleftarrow{f} B</math></b>	Mapping
107	<b><math>\leftrightarrow</math></b>	Equivalent
108	<b>A X B</b>	A cross B
109	<b>M X N</b>	M by n in matrices
110	<b><math>A^T</math></b>	Transpose of A
111	<b><math>A^{-1}</math></b>	Inverse of A
112	<b> A </b>	Determinant of A
113	<b><math>\leftrightarrow</math></b>	Line

114	$\overrightarrow{AB}$	Ray
115	$\overline{AB}$	Line segment
116	$[X]$	Step X
117	$\lceil X \rceil$	Ceiling of x
118	$\lfloor X \rfloor$	Floor of x
119	$T_r$	Trace
120	$\textcolor{blue}{y}$	Nu
121	$\nabla$	Velocity
122	$D$	Distance
123	$T$	Time
124	$\Psi$	Psi
125	$\rho$	Rho
126	$  $ $  $	Norm
127	$\emptyset$	Phi
128	$\therefore$	Since
129	$\therefore$	Therefore
130	$X$	Chi
131	$\uparrow$	Exponentiation
132	$\text{₹}$	Rupees symbol
133	$\text{₹}$	Dollar sign

## SQUARES, CUBES, SQUARE ROOTS

<b>n</b>	<b>n<sup>2</sup></b>	<b>n<sup>3</sup></b>	<b><math>\sqrt{N}</math></b>	<b>n</b>	<b>n<sup>2</sup></b>	<b>n<sup>3</sup></b>	<b><math>\sqrt{N}</math></b>
1	1	1	1.000	31	961	29791	5.568
2	4	8	1.414	32	1024	32768	5.657
3	9	27	1.732	33	1089	35937	5.745
4	16	64	2.000	34	1156	39304	5.831
5	25	125	2.236	35	1225	42875	5.916
6	36	216	2.449	36	1296	46656	6.000
7	49	343	2.646	37	1369	50653	6.083
8	64	512	2.828	38	1444	54872	6.164
9	81	729	3.000	39	1521	59319	6.245
10	100	1000	3.162	40	1600	64000	6.325
11	121	1331	3.317	41	1681	68921	6.403
12	144	1728	3.464	42	1764	74088	6.481
13	169	2197	3.606	43	1849	79507	6.557
14	196	2744	3.742	44	1936	85184	6.633
15	225	3375	3.873	45	2025	91125	6.708
16	256	4096	4.000	46	2116	97336	6.782
17	289	4913	4.123	47	2209	103823	6.856

18	324	5832	4.243	48	2304	110592	6.928
19	361	6859	4.359	49	2401	117649	7.000
20	400	8000	4.472	50	2500	125000	7.071
21	441	9261	4.583	51	2601	132651	7.141
22	484	10648	4.690	52	2704	140608	7.211
23	529	12167	4.796	53	2809	148877	7.280
24	576	13824	4.899	54	2916	157464	7.348
25	625	15625	5.000	55	3025	166375	7.416
26	676	17576	5.099	56	3136	175616	7.483
27	729	19683	5.196	57	3249	185193	7.550
28	784	21952	5.292	58	3364	195112	7.616
29	841	24389	5.385	59	3481	205379	7.681
30	900	27000	5.477	60	3600	216000	7.746

## 1.STATEMENTS

A Statement is a sentence which is either true or false but not both

1. A) NEGATION (~)

P	~P
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T	F
F	T

### B) DISJUNCTION ( $\vee$ )

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

### 2. CONJUNCTION ( $\wedge$ )

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

### 3. CONDITIONAL (IMPLICATION) ( $\Rightarrow$ )

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

### 4. BICONDITIONAL ( BI IMPLICATION ) ( $\Leftrightarrow$ )

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

### 5. CODITIONAL : $P \Rightarrow Q$

CONVERSE	:	$Q \Rightarrow P$
INVERSE	:	$\sim P \Rightarrow \sim Q$
CONTRA POSITIVE	:	$\sim Q \Rightarrow \sim P$

6.  $\sim(\sim P) \equiv P$

7. TAUTOLOGY : a)  $P \vee \sim P$   
b)  $(P \wedge \sim q) \Rightarrow P$

8. CONTRADICTION : a)  $P \wedge \sim q$   
b)  $(P \wedge \sim q) \vee (\sim P \vee q)$

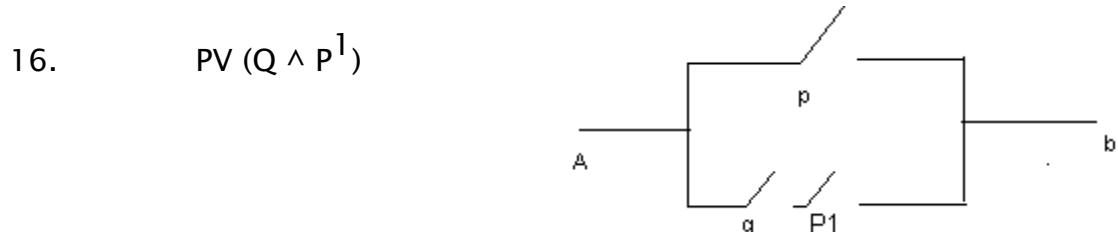
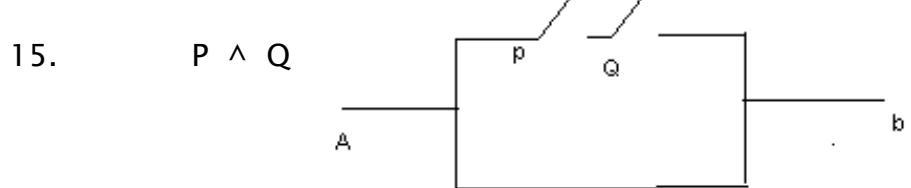
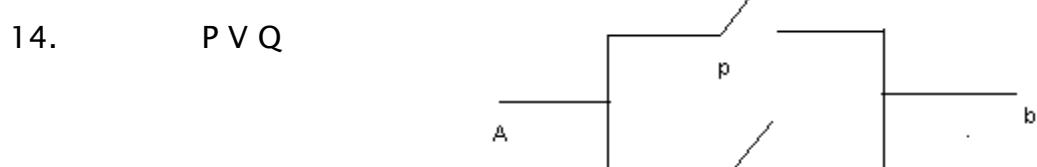
9.  $P \Rightarrow Q \equiv (\sim Q \Rightarrow \sim P)$

10.  $\sim(p \Rightarrow Q) \equiv P \wedge (\sim Q)$

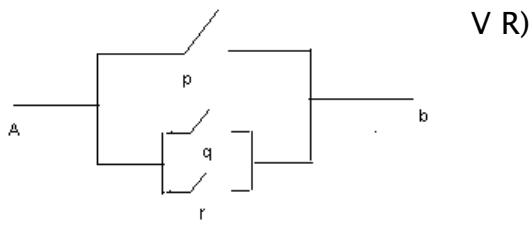
11.  $\sim(P \Leftrightarrow Q) \equiv (P \Leftrightarrow \sim Q) \equiv (\sim P \Leftrightarrow Q)$

12.  $\sim P \vee (P \wedge Q) \equiv (P \Rightarrow Q)$

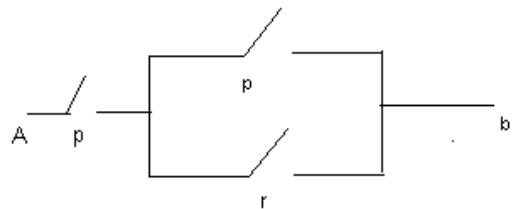
13.  $(P \Rightarrow Q) \wedge (Q \Rightarrow P) \equiv P \Leftrightarrow Q$



17.  $P \vee (Q \vee R)$



18.  $P \wedge (Q \vee R)$



### IDEMPOTENT LAWS:

20)  $P \vee P \equiv P$

21)  $P \wedge P \equiv P$

### COMMUTATIVE LAWS:

22)  $P \vee q \equiv q \vee P$

23)  $P \wedge q \equiv q \wedge p$

### ASSOCIATIVE LAWS:

24)  $(P \vee q) \vee r \equiv P \vee (q \vee r)$

25)  $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$

## DISTRIBUTIVE LAWS

$$26) p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$27) p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

## DEMORGAN LAWS

$$28) \sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$$

$$29) \sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$$

## IDENTITY LAWS

$$30) p \vee f \equiv p$$

$$31) p \wedge f \equiv f$$

$$32) p \wedge t \equiv p$$

$$33) p \vee t \equiv t$$

## COMPLEMENT LAWS

$$34) p \vee (\sim p) \equiv t$$

$$35) p \wedge (\sim p) \equiv f$$

$$36) \sim(\sim p) \equiv p$$

37)  $\sim t \equiv f$

38)  $\sim f \equiv t$

39) General form of an even number is  $2n$

40) General form of an odd number is  $(2n-1)$  or  $(2n+1)$

41) If  $x$  is odd then  $x^2$  is odd

42) If  $x$  is even then  $x^2$  is even

## I (b) Sets

Set is a well defined collection of objects.

1.  $A \cup B = \{x / x \in A \text{ or } x \in B\}$

2.  $A \cap B = \{x / x \in A \text{ and } x \in B\}$

3.  $A - B = \{x / x \in A \text{ and } x \notin B\}$

4.  $B - A = \{X / X \in B \text{ and } X \notin A\}$

5.  $A^1 = \{X / X \in \mu \text{ and } x \notin A\}$

6.  $(A^1) = \{X / X \in \mu \text{ and } x \in A\}$

7.  $A \Delta B = \{X / X \in A - B \text{ and } x \in B - A\}$

8.  $\emptyset = \{ \}$

9.  $A_1 \cup A_2 \cup \dots \cup A_n =$

10.  $A_1 \cap A_2 \cap \dots \cap A_n =$

11.  $\emptyset \subseteq A$
12.  $A \subseteq A$
13. IF  $A \subseteq B, B \subseteq A$  then  $A = B$
14. IF  $A \subseteq B, B \subseteq C$  then  $A \subseteq C$
15. IF  $A \subseteq B$ , then  $A \cup B = B$
16. IF  $A \subseteq B$ , then  $A \cap B = B$
17. IF  $A \subseteq B$ , then  $A - B = \emptyset$  AND  $B - A$  MAY OR MAY NOT BE  $\emptyset$
18.  $A \subseteq B$  then  $B^1 \subseteq A^1$

### INDEMPOTENT LAWS

19.  $A \cup A = A$
20.  $A \cap A = A$

### COMMUNTATIVE LAWS

21.  $A \cup B = B \cup A$
22.  $A \cap B = B \cap A$

### ASSOCIATIVE LAWS

23.  $A \cup (B \cup C) = (A \cup B) \cup C$
24.  $A \cap (B \cap C) = (A \cap B) \cap C$

### DISTIBUTIVE LAWS

25.  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
26.  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

### IDENTITY LAWS

27.  $A \cup \emptyset = A$

28.  $A \cap \emptyset = A$   
 29.  $A \cup A^1 = \mu$   
 30.  $A \cap A^1 = \emptyset$   
 31.  $(A^1)^1 = A$   
 32.  $\emptyset^1 = \mu$   
 33.  $\mu^1 = \emptyset$

#### DEMORAN LAWS

34.  $(A \cup B)^1 = A^1 \cap B^1$   
 35.  $(A \cap B)^1 = A^1 \cup B^1$   
 36.  $A \cap \emptyset = \emptyset$   
 37.  $A \cup \mu = \mu$   
 38.  $A - (B \cup C) = (A - B) \cap (A - C)$   
 39.  $A - (B \cap C) = (A - B) \cup (A - C)$   
 40.  $A \Delta B = (A - B) \cup (B - A)$   
 $= (A - B) \cap (B - A)$   
 41. IF A,B are disjoint then  $A \cap B = \emptyset$   
 42.  $(A \cup B \cup C)^1 = A^1 \cap B^1 \cap C^1$   
 43.  $A - A = \emptyset$   
 44.  $A - \emptyset = A$   
 45.  $\emptyset = A = \emptyset$   
 46. if  $A \cap B = \emptyset$  then  $A - B = A$   
 47. if  $A \cap B = \emptyset$  then  $B - A = B$   
 48. if  $A \cap B = \emptyset$  then  $A \subset B^1$  and  $B \subset A^1$   
 49. if  $A \cup B = \emptyset$  then  $A = \emptyset$  and  $B = \emptyset$   
 50.  $A - (A - B) = A \cap B$

51.  $A \cup (A - B) = A$   
 52.  $A \cup (B - A) = A \cup B$   
 53.  $A \cap (B - A) = \emptyset$   
 54. if  $A - B = \emptyset$  then  $A \cap B = A$  and  $A \subset B$   
 55.  $A \cap B^1 = A - B$   
 56.  $A^1 \cap B = B - A$   
 57.  $A - B^1 = A \cap B$   
 58.  $B - A^1 = A \cap B$   
 59.  $A^1 - B^1 = B - A$   
 60. if  $A - B = \emptyset$  and  $B - A = \emptyset$  then  $A = B$   
  
 61. if  $A \cap B = A$  and  $A \cup B = B$  then  $A \subset B$   
 62. if  $A > B$  then  $A^1 - B^1 = \emptyset$   
 63.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$   
 64.  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$   
 65. if  $A, B$  are disjoint then  $n(A \cup B) = n(A) + n(B)$   
 66. if  $A, B, C$  are disjoint with each other then  
 $n(A \cup B \cup C) = n(A) + n(B) + n(C)$   
 67.  $n(A - B) = n(A) + n(A \cap B)$   
 68.  $n(B - A) = n(B) + n(A \cap B)$   
 69. no. of subsets =  $2^n$   
 70. no. of elements in a power set =  $n[P(A)] = 2^n$   
 71. no. of proper subsets =  $2^n - 1$

72. For infinite set, no.of subsets are infinite .

73. Cartesian Product:

$$AXB = \{(a, b) / a \in A, b \in B\}$$

74. if  $A = \emptyset$  or  $B = \emptyset$  then  $AXB = \emptyset$

75. if  $n(A) = P, n(B) = Q$  then  $n(AXB) = PQ$

76.  $AXB \neq BXA$

77.  $n(AXB) = n(BXA) = n(A) \times n(B)$

78. if  $A \subseteq B$  then  $AXB \subseteq BXA$

79.  $AXB(B \cup C) = (AXB) \cup (AXC)$

80.  $AXB(B \cap C) = (AXB) \cap (AXC)$

81.  $AXB(B - C) = (AXB) - (AXC)$

82.  $A \subseteq B, C \subseteq D$  then  $AXB \subseteq BXD$

83.  $(AXB) \cup (CXD) \subseteq (A \cup C) \times (B \cup D)$

84.  $(AXB) \cap (CXD) \subseteq (A \cap C) \times (B \cap D)$

85.  $A_1 \times A_2 \times \dots \times A_n = \prod_{i=1}^n A_i$

86.  $n(A) = \text{cardinal number} =$

*no.of distinct elements in a set*

87. if  $n(A) =$

$n(B)$  then  $A, B$  then  $A, B$  are equivalent sets

88. if  $A, B$  have the same elements then  $A, B$  are equal sets

89.  $n(A \cap B^1) = n(A) - n(A \cap B)$

90.  $n(B \cap A^1) = n(B) - n(A \cap B)$

91.  $\mu \Delta \emptyset = \mu$

92. The Minimum possible value of  $n(A \Delta B)$  is  $n(B) - n(A)$

## 2 (a) Relations

Let A,B be two non empty sets A relations from A to B is a subset of AXB. i.e:  $R \subseteq AXB$

- 1) If  $n(A) = P, N(B) = q$  then no of relations =  $2^{pq}$
- 2) If A,B are two sets and R is a relation from A to B then domain of  $R=\{x/(x,y)\in R\}$  range of R= {  $y/(x,y) \in R\}$
- 3) no of subsets of AXB =  $2^{pq}$
- 4)  $R = AXA$  is called universal relation
- 5)  $R= \{(a,a)/a\in A\}$  is called identify relation
- 6)  $R=\emptyset$  is called void relation.
- 7) If  $n(A) = m$  then  $n(AXA) = m^2$
- 8) If  $n(A) = m$  then no. of relations=  $m^2$
- 9) If  $n(A) = 0$  then no. of relations = 1
- 10) If  $(a,a) \in R, \forall a \in A$  is called reflexive relation
- 11) If  $(a,b) \in R$  then  $(b,a) \in R$  then R is systematic relation.
- 12) If  $(a,b) \in R, (b,c) \in R$  then  $(a,c) \in R$  then R is transitive relation.
- 13) R is an equivalence relation if
  - a)  $a R a, \forall a \in A$
  - b)  $a R b \Rightarrow b R a$
  - c)  $a R b, b R c \Rightarrow a R c$
- 14) If  $(a,b) \in R, (b,a) \in R$  then  $a = b$  is an anti symmetric relation.
- 15) If  $R = \{(x,y) / x \in A, Y \in B\}$  then

$$R^{-1} = \{ (y, x) / (x, y) \in R \}$$

- 16) The universal relation on a non empty set is always reflexive, symmetric and transitive.
- 17) The identity relation on a non empty set is always reflexive, symmetric and transitive
- 18) The identity relation on a non empty set is always anti symmetric.
- 19) If  $R$  is said to be a partial order relation if it is reflexive, anti symmetric and transitive.
- 20) No. of reflexive relations in a set containing ' $n$ ' elements is  $2^{n^2-n}$

## 2(B) FUNCTION

If  $f: A \rightarrow B$  is said to be a function if every element in  $A$  has unique image in  $B$

IF  $f: A \rightarrow B$  is a function if

(i)  $x \in A \Rightarrow f(x) \in B$

(ii)  $x_1, x_2 \in A, x_1, x_2 \Rightarrow f(x_1) = f(x_2)$

1) If  $n(A) = P, n(B) = q$  then no of functions =  $q^P$

2) if  $f: A \rightarrow B$  is a function then  $A$  is called domain and  $B$  is called co domain

3) if  $f: A \rightarrow B$  is a function then

$$\text{Range} = f(A) = \{f(x) / x \in A\}$$

4) If  $A \subseteq R$  then  $f: A \rightarrow B$  is called real variable function.

5) If  $B \subseteq R$  then  $f: A \rightarrow B$  is called real valued function

6) If  $A \subseteq R, B \subseteq R$  then  $f: A \rightarrow B$  is called real function

7)  $(f + g)(x) = f(x) + g(x)$

8)  $(f - g)(x) = f(x) - g(x)$

$$9) (f \circ g)(x) = f(x) \cdot g(x)$$

$$10) (f/g)(x) = f(x) / g(x)$$

$$11) \left(\frac{f}{g}\right)(x) = f(x)/g(x)$$

$$12) f^n(x) = [f(x)]^n, n \in \mathbb{Z}^+$$

$$13) (f + k)(x) = f(x) + K, K \in \mathbb{R}$$

14) Range c codomain

$$15) f \subseteq A \times B$$

16) A function  $f$  is said to be periodic with period 'K' if  $f(x), \forall x \in R$

17)  $f$  is bounded  $\exists k_1, k_2$  such that  $k_1 \leq f(x) \leq k_2$

18) If  $f : A \rightarrow B$  is one one then

$$\text{a)} f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \\ \text{(or)}$$

$$\text{b)} x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

19) If  $f : A \rightarrow B$  is one one, A,B are finite then  $n(A) \leq n(B)$

20) If  $f : A \rightarrow B$  is one one then no.of one one functions.

$$= n(B)P_{n(A)}, \text{if } n(B) \geq n(A)$$

$$= 0, \text{if } n(B) < n(A)$$

21) If  $f : A \rightarrow B$  is on to  $\exists Y \in B$  such that  $f(x) = y$

22) If  $n(A) = p, n(B) = 2$  then no.of onto functions =  $2^p - 2$ , if  $p \geq 2$

$$= o \quad , if p < 2$$

23) If  $n(A)=p, n(B)=q$  then no.of onto functions  $= \sum_{r=1}^q (-1)^{q-r} qc_r \times (r)^p, if p \geq q$

$$= o \quad , if p < q$$

24) If  $f : A \rightarrow B$  is one one and onto then it is bijection

25) if  $n(A) = P, n(B) = P$  then no.of bijections  $= P!$

26) If  $n(A) \neq n(B)$  then no.of bijections  $= 0$

27) If  $n(A) = P, f : A \rightarrow A$  then

a) no.of functions  $= p^p$

b) no.of one one functions  $= P!$

c) no.of onto functions  $= \sum_{r=1}^q (-1)^{p-r} pc_r (r^p)$

d) no.of bijections  $= P!$

28) If  $f : A \rightarrow B, g : B \rightarrow C$  are one one functions then go  $f : A \rightarrow C$  is also one one.

29) if  $f : A \rightarrow B, g : B \rightarrow C$  are onto functions then go  $f : A \rightarrow C$  is also onto.

30) If  $f : A \rightarrow B, g : B \rightarrow C$  are bijections then go  $f : A \rightarrow C$  is also bijection

31) If  $f : A \rightarrow B, g : B \rightarrow C$ , if  $gof$  is one one then  $f$  is one one

32) If  $f : A \rightarrow B, g : B \rightarrow C$ . If  $gof$  is onto then  $g$  is onto

- 33) If  $f$  is bijection then  $f^{-1}$  exists
- 34)  $I: A \rightarrow A, I(x) = x$  is called identify function.
- 35)  $f : A \rightarrow B, f \circ I_A = I_{B^0} f = f$
- 36)  $f : A \rightarrow B$ , is bijection then  $f^{-1} \circ f = I_A, f \circ f^{-1} = I_B$
- 37)  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$
- 38) If  $f : A \rightarrow B, g : B \rightarrow C, h : C \rightarrow D$  then  $h \circ (g \circ f) = (h \circ g) \circ f$
- 39)  $f(x) = K$  is called constant function and no.of constant functiond = n (B)
- 40)  $f^{-1}\{(b, a)/(a, b) \text{ } \epsilon f\}$ , where  $f$  is bijective
- 41) If  $f : A \rightarrow B$ , is bijective,  $f(a) = b$  then  $a = f^{-1}(b)$
- 42) Range of constant function is singleton set
- 43) If  $f : A \rightarrow B, g : B \rightarrow C$ , then
- $f, g$  are injective  $\Rightarrow g \circ f$  is injective
  - $f, g$  are surjective  $\Rightarrow g \circ f$  is surjective
  - $f, g$  are bijective  $\Rightarrow g \circ f$  is bijective
- 44)  $f \circ g \neq g \circ f$
- 45) If  $f(x) = \sqrt[n]{a - xn}$  then  $f \circ f(x) = x$
- 46) If  $y = f(x) = \frac{Ax+B}{Cx-A}$  then  $f(y) = x$
- 47) If  $f(x) = \frac{a^x + a^{-x}}{2}$  then  $f(x+y) + f(x-y) = 2 f(x) \cdot f(y)$

48) If  $S$  is a set having ' $n$ ' elements, then the no.of binary operations that can be defined in  $S$  is  $n^{n^2}$  and operations which are commutative is  $n \frac{n(n+1)}{2}$

49)  $f(x) = (x)$ , where  $f(x) = n$  is an integer such that  $n \leq x < n+1$  is called a step function

50)  $0 \leq x - (x) < 1$

51)  $\lfloor x \rfloor$  is called floor of  $f$

eg:  $\lfloor 7.5 \rfloor = 7$

52)  $\lceil x \rceil$  is called ceiling of  $f$

eg:  $\lceil 7.5 \rceil = 8$

53)  $\lfloor x \rfloor = \lceil x \rceil$  if  $x \in Z$

54)  $\lfloor x \rfloor + 1 = \lceil x \rceil$ , if  $x \notin R - Z$

55)  $x \in [a, b] \Rightarrow a \leq x \leq b$

56)  $x \in (a, b) \Rightarrow a < x < b$

57)  $x \in [a, b) \Rightarrow a \leq x < b$

58)  $x \in (a, b] \Rightarrow a < x \leq b$

59)

Function	Inverse
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$x$	$x$
$n x$	$\frac{x}{n}$
$ax+b$	$\frac{x-b}{a}$
$\frac{1}{x}$	$\frac{1}{x}$
$\log_e x$	$e^x$
$e^x$	$\log_e x$
$a \sin bx$	$\frac{1}{b} \sin^{-1} \frac{x}{a}$
$\frac{ax+b}{cx+d}$	$- \frac{(b-dx)}{a-cx}$
$\sqrt[n]{x^2 + 1}$	$\sqrt[n]{x^n - 1}$

- 60)  $[a, \infty) = \{ x \in R / x \geq a \}$
- 61)  $(a, \infty) = \{ x \in R / x > a \}$
- 62)  $(-\infty, a) = \{ x \in R / x < a \}$
- 63)  $(-\infty, a) = \{ x \in R / x \leq a \}$
- 64)  $(-\infty, \infty) = R$
- 65) If  $f(-x) = f(x)$  then  $f$  is an even function.
- 66) If  $f(-x) = -f(x)$  then  $f$  is an odd function.
- 67)  $f(x) + f(-x)$  is an even function.
- 68)  $f(x) - f(-x)$  is an odd function.
- 69) The product of two even functions is an even function.

70) The product of two odd functions is an even function.

71) The product of one even and one odd function is an odd function.

72)

Even functions	$\sec x, \cos x, 2^{-x^2}, \frac{a^x + a^{-x}}{2}, x^{2n}, \sqrt{1+x^2}, \frac{1}{x^{2n+1}}$
odd functions	$\sin x, \tan x, \frac{ x }{x}, \forall x \in \mathbb{R} - \{0\}, x^{2n+1}, \operatorname{cosec} x, \cot x$

73)  $f(x) = c$       R       $\{c\}$

74)  $f(x) = x$       R      R

75)  $f(x) = |x|$       R       $\mathbb{R}_+ \cup \{0\}$

76)  $f(x) = ax+b$       R      R

77)  $f(x) = x^2$       R       $\mathbb{R}_+ \cup \{0\}$

78)  $f(x) = \frac{1}{x}$        $\mathbb{R} - \{0\}$        $\mathbb{R} - \{0\}$

79)  $f(x) = x^3$       R      R

80)  $f(x) = \sqrt{1-x^2}$        $[-1, 1]$        $[0, 1]$

81)  $f(x) = \sin x$       R       $[-1, 1]$

82)  $f(x) = \cos x$       R       $[-1, 1]$

83)  $f(x) = \tan x$        $\mathbb{R} - \{x/x=(2n+1)\pi/2, n \in \mathbb{Z}\}$       R

84)  $f(x) = \operatorname{wt} x$        $\mathbb{R} - \{x/x=n\pi, n \in \mathbb{Z}\}$       R

85)  $f(x) = \sec x$        $\mathbb{R} - \{x/x=(2n+1)\pi/2, n \in \mathbb{Z}\}$        $(-\infty, -1] \cup [1, \infty)$

86)  $f(x) = \cos cx$        $\mathbb{R} - \{x/x = n\pi, n \in \mathbb{Z}\}$        $(-\infty, -1] \cup [1, \infty)$

87)  $f(x) = ax$       R       $(0, \infty)$

88)  $f(x) = \log_a x$        $(0, \infty)$       R

$$89) \quad f(x) = \sqrt{x} \quad [0, \infty) \quad [0, \infty)$$

$$90) \quad f(x) = [x] \quad \mathbb{R} \quad \mathbb{Z}$$

91) In the graph of a relation if the vertical line does not intersect the graph at more than one point then it is a function.

92) In the graph of a function if the horizontal line does not intersect the graph at more than one point, then it is a one one function.

93) In the graph of constant function the line joining the points is parallel to axis

94) in the graph of identity function the joining points passing through origin

### (3) POLYMINALS

An algebraic expression of the form

$ax^n + x^{n-1} + a_2x^{n-2} + \dots + (a_n \text{ constant}), a_0 \neq 0, n \in N$  is called a polynomials of degree n.

1) The set of polynomials is

- i. Closed under addition and multiplication.
- ii. Commutative under addition and multiplication.
- iii. Associative under addition and multiplication.
- iv. Additive identity is 0.
- v. Multiplicative identity is 1.
- vi. Distributive property holds.

2) 1<sup>st</sup> degree polynomial is  $ax+b$

3) 2<sup>nd</sup> degree polynomial is  $ax^2 + bx + c$

4) Zero of  $ax+b$  is  $-b/a$ .

5) Zero of  $ax-b$  is  $b/a$ .

6) Dividend = (divisor x quotient) + remainder,

$$7) \quad (a + b)^2 = a^2 + 2ab + b^2$$

$$8) \quad (a - b)^2 = a^2 - 2ab + b^2$$

- 9)  $(a + b)(a - b) = a^2 - b^2$   
 10)  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$   
 11)  $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2ab + 2ac + 2ad + 2bc + 2bd + 2cd$   
 12)  $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$   
 13)  $(a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$   
 14)  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$   
 15)  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$   
 16)  $a^4 - b^4 = (a + b)(a - b)(a^2 + b^2)$   
 17)  $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$   
 18)  $(x + a)(x + b)(x + c) = x^2 + (a + b)x + ab$   
 19)  $(ax + b)(cx + d) = acx^2 + (ac + bd)x + bd$   
 20)  $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$   
 21)  $(a + b)^4 = a^4 + b^4 + 6a^2b^2 + 4a^3b + 4ab^3$   
 22)  $(a - b)^4 = a^4 + b^4 + 6a^2b^2 - 4a^3b - 4ab^3$

23)

### Homogeneous expressions

Variables	Degree	Standard form
$x, y$	1	$ax + by$
$x, y$	2	$ax^2 + bxy + cy^2$
$x, y$	3	$ax^3 + bx^2y + cxy^2 + dy^3$
$x, y, z$	1	$ax + by + cz$

$$x, y, z \quad 2 \quad ax^2 + by^2 + cz^2 + dxy + eyz + fzx$$

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$$x, y, z \quad 3 \quad ax^3 + by^3 + cz^3 + dx^2y + exy^2 + fy^2z + gyz^2 + hz^2x + kzx^2 + lxyz$$

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24) An expression is said to be symmetric if  $f(x, y) = f(y, x)$

25) The sum of two symmetric functions is also symmetric

26) The difference of two symmetric functions is also symmetric

27) The product of two symmetric functions is also symmetric

28) The quotient of two symmetric functions is also symmetric

29) An expression  $f(x, y, z)$  is said to be a cyclic expression if  $f(x, y, z) = f(y, z, x)$

$$30) \left( \sum_{a,b,c} a \right) = a + b + c$$

$$31) \prod_{a,b,c} a = abc$$

32)  $ax^2 + bx + c = 0, a \neq 0$  is called quadratic equation.

$$33) ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

34) Quadratic equation with roots  $\alpha, \beta$  is  $x^2 - (\alpha + \beta)x + \alpha\beta = 0$  (or)  $(x - \alpha)(x - \beta) = 0$

35) The roots of  $\alpha, \beta$  of  $ax^2 + bx + c = 0$

- i) If  $b^2 - 4ac = 0$  then roots are real and equal.
- ii)  $b^2 - 4ac > 0$  then roots are real and distinct

iii)  $b^2 - 4ac < 0$  then roots are imaginary and distinct

<u>CONDITIONS</u>	<u>ROOTS</u>
36) $b^2 - 4ac$ is a perfect square	Roots are rational
37) $b = 0$	Roots are equal but opposite in sign
38) $c = 0$	Roots are reciprocal
39) $c = 0$	One root is zero
40) $b = 0, c = 0$	Both roots are zero
41) $a = 0$	One root is infinity
42) $b = 0, c = 0$	Both roots are infinity
43) $a = -c$	Roots of opposite signs and reciprocal
44) $a, b, c$ have same sign	Both roots negative
45) $a, c$ have same sign $2b$ has opposite sign	Both roots are positive.

46) If  $\alpha, \beta$  are roots of  $ax^2+bx+c=0$  then

$$a) \alpha + \beta = -\frac{b}{a} \quad \alpha \beta = \frac{c}{a}$$

$$b) \alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$$

$$c) \alpha^2 - \beta^2 = \frac{\pm b\sqrt{b^2 - 4ac}}{a^2}$$

$$d) \alpha^3 + \beta^3 = \frac{3abc - b^3}{a^3}$$

$$e) \alpha^4 + \beta^4 = \frac{b^4 - 4ab^2c + 2a^2c^2}{a^4}$$

$$f) \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{-b}{\sqrt{ac}}$$

$$g) \frac{\alpha+\beta}{\alpha^{-1} + \beta^{-1}} = \frac{c}{a}$$

$$h) \alpha^2\beta + \alpha\beta^2 = \frac{-bc}{a^2}$$

<u>ROOTS</u>	<u>AUADRATIC EQUATION</u>
47) $\frac{a}{b}, \frac{b}{a}$	$abx^2 - (a^2 + b^2)x + ab = 0$
48) $\frac{a+b}{2}, \frac{a-b}{2}$	$4x^2 - 4ax + (a^2 - b^2) = 0$
49) $a, \frac{1}{a}$	$a(x^2 + 1) = x(a^2 + 1)$
50) $\frac{a+b}{a}, \frac{a+b}{b}$	$abx^2 - (a+b)^2 x + (a+b)^2 = 0$
51) $\frac{a-b}{b}, \frac{b-a}{a}$	$abx^2 - (a-b)^2 x + (a-b)^2 = 0$
52) $\frac{a+b}{a-b}, \frac{a-b}{a+b}$	$(a+b)x^2 - 2(a^2 + b^2)x + 1 = 0$
53) $\frac{c(a-b)}{a(b-c)}, 1$	$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

54) If a,  $x^2 + b_1x + c_1 = 0$ ,  $a_2x^2 + b_2x + c_2 = 0$  have a common root then common root is  $\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$  (or)  $\frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$

55) If are root of  $ax^2 + bx + c = 0$  is twice other then  $2b^2 = 9ac$

56) If the roots of  $ax^2 + bx + c = 0$  are in the ratio m:n the  $(m+n)^2 ac = mn b^2$

57) If the roots of  $x^2 - px + q = 0$  are differed by "s" then  $p^2 + 4q^2 = (1 + 2q)^2$

58) If the roots of the equation  $x^2 - px + q = 0$  are consecutive numbers then  $p^2 = 4q + 1$

59) If the roots of  $lx^2 + nx + n = 0$  be in the ratio p:q then  $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

60) If a rational integral function of x, say f(x) is divided by x-a then the remainder is f(a)

61) If f(x) is a rational integral function of x and f(a)=0 then x-a is a factor of f(x)

62) Velocity =  $\frac{\text{distance}}{\text{time}}$

63)  $1\text{m/sec} = \frac{18}{5} \text{ km / hour}$

64)  $1\text{km/hour} = \frac{5}{18} \text{ m / sec}$

65) If  $ax^2 + bx + c = 0$  ( $a > 0$ ) has real roots  $r_1, r_2$  then

$$x < r_1, ax^2 + bx + c > 0$$

$$x = r_1, ax^2 + bx + c = 0$$

$$r_1 < x < r_2, ax^2 + bx + c < 0$$

$$x = r_2, ax^2 + bx + c = 0$$

$$x > r_2, ax^2 + bx + c > 0$$

66) If  $ax^2 + bx + c = 0$  ( $a > 0$ ) has equal real solutions  $r_1 = r_2 = r$  then

$$x < r, ax^2 + bx + c > 0$$

$$x = r, ax^2 + bx + c = 0$$

$$x > r, ax^2 + bx + c > 0$$

67) If  $ax^2 + bx + c = 0$  ( $a > 0$ ) has non real solutions then  $ax^2 + bx + c > 0, \forall x \in r$

68) If  $(x-\alpha)(x-\beta) < 0, \alpha < \beta$  then  $\alpha < x < \beta$

69) If  $(x-\alpha)(x-\beta) > 0, \alpha < \beta$  then  $x > \beta$  or  $x < \alpha$

70) **MATHEMATICAL INDUCTION:** If  $P(n)$  is a statement which (i) true for  $n=1$  and is

(ii) True for  $n+1$  whenever it is true for  $1, 2, \dots, n$ , then the statement  $P(n)$  is true for all natural numbers  $n$ .

71)  $2^n > n \forall n \in N$

72) 3 is a factor of  $2^{2n} - 1 > n \forall n \in N$

73) If  $n$  is odd,  $n(n^2 - 1)$  is divided by 24

74)  $x^n - y^n$  is divisible by  $x - y$

75) If  $n$  is odd,  $x^n + y^n$  is divisible by  $x + y$

76)  $2^n > 2n$

77)  $(n)^{n+1} > (n+1)^n, n \geq 3$

78)  $2^n > 2n + 1, n \geq 3$

79)  $2^n > n^2, n \geq 5$

80)  $3^n \geq 2n + 1$

81)  $(1+x)^n \geq 1 + nx$

82)  $x^{2n+1} + y^{2n+1}$  is divisible by  $x + y$

- 83)  $n^5 - n$  is divisible by 30,  $n > 1$
- 84) If  $P$  is a natural number then  $p^2 + p + 1$  is a factor of  $p^{n+1} + (p + 1)^{2n-1}$
- 85) If  $a, b, c$  are 3 consecutive integers then  $a^3 + b^3 + c^3$  is divisible by 9.
- 86) If  $x_1 \cdot x_2 \cdot x_3 \cdots \cdots \cdot x_n = 1 \Rightarrow x_1 + x_2 + \cdots + x_n \geq n$  ( $n \geq 2$ )
- 87)  $n^p - n$  is divided by  $P$  if  $P$  is a prime,  $n > 1$ .
- 88)  $(2n + 1)^{2n}$  is divided by 8 gives remainder 1
- 89)  $n(n+1)(n+2)$  is divisible by 6
- 90)  $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}, \forall n \geq 2$
- 91) In general 3 consecutive numbers can be taken as  $n, n+1, n+2$
- 92) If  $x^2 + ax + b = 0, x^2 + px + q = 0$  have common root then common root is  $\frac{b-q}{p-a}$  (or)  $\frac{aq-bp}{b-q}$
- 93) If  $a_1 x^2 + b_1 x + c_1 = 0, a_2 x^2 + b_2 x + c_2 = 0$  have same root then  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- 94)  $n! = n(n-1)(n-2) \dots \cdot 3 \cdot 2 \cdot 1$
- 95)  $nc_r = \frac{n!}{(n-r)!r!}$
- 96)  $np_r = \frac{n!}{(n-r)!}$
- 97)  $nc_0 = nc_n = 1$
- 98)  $nc_1 = nc_{n-1} = n$
- 99)  $nc_r = nc_{n-r}$
- 100) If  $nc_p = nc_q$  then  $p + q$  (or)  $p = q$
- 101)  $0! = 1$
- 102)  $1! = 1$
- 103)  $nc_{r-1} + nc_r = (n+1)c_r$
- 104)  $\frac{nc_r}{nc_{r-1}} = \frac{n-r+1}{r}$
- 105) Binomial expansion  

$$(x+y)^n = nc_0 x^n + nc_1 x^{n-1}y + nc_2 x^{n-2}y^2 + \cdots + nc_r x^{n-r}y^r + \cdots + nc_n y^n. (n \in N)$$
- 106) General term,  $tr_{r+1} = nc_r x^{n-r}y^r$
- 107) No. of terms in  $(x+y)^n$  is  $n+1$
- 108) No. of terms in  $(x_1 + x_2 + \cdots + x_n)^n$  is  $(n+r-1)c_{r-1}$
- 109) Sum of the coefficient in the expansion of  $(x+y)^n$  is  $2^n$
- 110) In  $(x+y)^n, n$  is even then  $(\frac{n}{2} + 1)^{th}$ , term is middle term
- 111) In  $(x+y)^n, n$  is odd, then  $(\frac{n+1}{2})^{th}, (\frac{n+3}{2})^{th}$  terms are middle terms.

112) numerically greatest terms in  $(1+x)^n$  is  $\frac{1x_1(n+1)}{1x_1+1} = p + F$ ,  $0 < F < 1$  then  $(p+1)^{th}$  terms is numerically greatest.

113) The independent term in  $(x + \frac{1}{x})^{2n}$  is  $\frac{(2n-1)!}{n!} 2^n$

114) In  $(x+y)^n$  if n is even, greatest.

115) In  $(1+x)^n$ , sum of even binomial coefficients =  $2^n - 1$

116) In  $(1+x)^n$ , sum of odd binomial coefficients =  $2^n - 1$

117) Sum of the coefficients of  $(a_1 x_1 + a_2 x_2 + \dots + a_n x_n)^m$

118) The term independent of x in  $(ax^p + \frac{b}{x^q})^n$  is  $t_{r+1}$  where  $r = \frac{np}{p+q}$

119)  $np_n = np_{n-1} = n!$

120)  $\frac{npr}{np_{n-1}} = n - r + 1$

121) The total no. of factors of  $p_1^{n_1} \cdot p_2^{n_2} \dots \cdot p_r^{n_r}$  is given by  $(n_1 + 1) \times (n_2 + 1) \times \dots \times (n_r + 1)$

122) If  $N = p_1^{n_1} \cdot p_2^{n_2} \dots \cdot p_r^{n_r}$  then sum of the factors  
 $= \frac{(p_1^{n_1+1})}{p_1-1} \times \frac{p_2^{n_2+1}-1}{p_2-1} \times \dots \times \frac{p_r^{n_r+1}-1}{p_r-1}$

123)  $f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$  and if

i)  $a_0 + a_1 + a_2 + \dots + a_n = 0$

124) Coefficient of  $x^n$  in  $(x+1)(x+2) \dots (x+n)$  is 1

125) Coefficient of  $x^{n-1}$  in  $(x+1)(x+2) \dots (x+n)$  is  $\frac{np-r}{p+q} + 1$

126) In  $(ax^p + \frac{b}{x^q})^n$ , term containing  $x^r$  is  $\frac{np-r}{p+q} + 1$

127) If the speed of a boat in still water be x km/hr and the speed of the stream be y km/hr then.

i) Speed of the boat with the stream (downstream) =  $(x+y)$  km/hr

ii) Speed of the boat against the stream (upstream) =  $(x-y)$  km/hr

128) Man's rate in Stillwater =  $1/2$  [Man's rate with the current + man's rate against the current]

129) Rate of current =  $1/2$  [Man's rate with the current - man's rate against the current]

130)  $a + b = \sqrt{(a-b)^2 + 4ab}$

131)  $a-b = \sqrt{(a+b)^2 - 4ab}$

132) If  $x + \frac{1}{x} = a$  then  $x - \frac{1}{x} = \sqrt{a^2 - 4}$

133) If  $x - \frac{1}{x} = a$  then  $x + \frac{1}{x} = \sqrt{a^2 + 4}$

134) If  $x + \frac{1}{x} = 0$  then  $x - \frac{1}{x}$  is not exists

135) If  $x - \frac{1}{x} = 0$  then  $x + \frac{1}{x} = \pm 2$

136)  $nc_0 + nc_1 + nc_2 + \dots + nc_n = 2^n$

137)  $nc_0 + nc_2 + nc_4 + \dots + nc_1 + nc_3 + \dots = 2^{n-1}$

138)  $y = mx^2$  is a parable with vertex (0,0)

139)  $y = mx^2$  is a symmetric about y axis ( $\varepsilon q_1, q_2$ )

140)  $x = my^2$  is a symmetric about x axis ( $\varepsilon q_1, q_4$ )

141)  $x = -my^2$  is symmetric about x axis ( $\varepsilon q_2, q_3$ )

142)  $y = -mx^2$  is symmetric about y axis ( $\varepsilon q_3, q_4$ )

143) If one root of  $ax^2 + bx + c = 0$  is / then other root is c/a

144) If  $nc_{r-1}, nc_r, nc_{r+1}$  are in A.P then

145) The sum of coefficient of odd powers of x in  $[f(x)]^n$  is  $\frac{[f(1)]^n - [f(-1)]^n}{2}$

146) The sum of coefficient of even power of x in  $[f(x)]^n$  is  $\frac{[f(1)]^n + [f(-1)]^n}{2}$

147) If a,b,c are 3 consecutive coefficient of  $(1+x)^n$  then  $n = \frac{2ac+b(a+c)}{b^2-ac}$ ,  $r = \frac{a(b+c)}{b^2-ac}$

## (4) LINEAR PROGRAMMING

1) **CONVEX SET**:- If  $p_1 q \in X \Rightarrow pq \in X$  then  $X$  is convex.

2) If  $(x, y) \in q_1 \Rightarrow x > 0, y > 0$

3) If  $(x, y) \in q_2 \Rightarrow x < 0, y > 0$

4) If  $(x, y) \in q_3 \Rightarrow x < 0, y < 0$

- 5) If  $(x, y) \in q_4 \Rightarrow x > 0, y < 0$
- 6) Equation of x axis is  $y=0$
- 7) Equation of y axis is  $x=0$
- 8) The line  $x=k$  is parallel to y axis
- 9) The line  $y=k$  is parallel to x axis
- 10) The line  $x=k$  is perpendicular to x axis
- 11) The line  $y=k$  is perpendicular to y axis
- 12) Midpoint of  $(x_1, y_1), (x_2, y_2)$  is  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- 13) The distance between  $(x_1, y_1), (x_2, y_2)$  is  $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$
- 14) The line  $y = mx+c$  with slope 'm' and passing through  $(0,c)$
- 15) **OBJECTIVE FUNCTION:-** The expression  $f = ax + by$  where  $a, b \in \mathbb{R}$  which is to be maximized (or) minimized is called the objective function.
- 16) **LPP:-** A LPP consists of minimizing (or) maximizing objective function  $f = ax + by$   $a, b \in \mathbb{R}$  subject to certain constraints expressible as linear in equations in x and y.
- 17) A closed convex polygon is the set of all points within and on some polygon with a finite number of vertices.
- 18) An open convex region is the set of all points within and on some polygon, which is open on one side, with a finite number of vertices.
- 19) **FUNDAMENTAL THEOREM:-** If the values of the expression  $f = ax + by$  are considered over the set of points constituting a non empty closed convex polygon then maximum value (or)

minimum value of  $f$  occurs on at least one of the vertices of the polygon.

20) **ISO PROFIT LINE:-** Any line belonging to the system of parallel lines given by the objective function for various values of the objective function  $f$ , is called an ISO profit line.

21) If ISO profit lines moving away from the origin then  $f$  will be increased.

22) **FEASIBLE REGION:-** The solution set of constraints of a linear programming problem is a convex set (open (or) closed) called the feasible region.

23) **FEASIBLE SOLUTION:-** Any point  $(x,y)$  in the feasible region gives solution to LPP called a feasible solution.

24) If the ISO profit line coincides with the side of the convex polygonal region the problem has infinite solution.

25) If inclination of a line ' $\theta$ ' then slope =  $\tan \theta$

26) The solution set of linear equations is a set of points called polyhedral set.

## (5) REAL NUMBERS

1)  $a^m = a \times a \times a \times \dots \dots \dots \times a$ ,  $m$  Times

2)  $a^m \times a^n = a^{m+n}$

3)  $a^m \div a^n = a^{m-n}$

4)  $(a - b)^m = a^m \times b^m$

5)  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$  ( $b \neq 0$ )

$$6) a^0 = 1$$

$$7) (a^m)^n = a^{mn}$$

$$8) a^{-n} = \frac{1}{a^n} (a \neq 0)$$

$$9) a^{m-n} = \frac{1}{a^{n-m}} \text{ if } n > m (a \neq 0)$$

$$10) \text{ If } a^m = a^n \text{ then } m = n (a \neq 0, a \neq 1)$$

$$11) a^m = a^n \Rightarrow \text{either } a = 1 \text{ or } m = n$$

$$12) a^m = b^m \Rightarrow \text{either } m = 0 \text{ or } a = b$$

$$13) \sum b - c = 0$$

$$14) \sum a(b - c) = 0$$

$$15) \text{ If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$$

$$16) \text{ If } \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$$

$$17) \text{ If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

$$18) \text{ If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$$

$$19) \text{ If } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$20) \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{d} = \frac{c}{a}$$

$$21) \text{ If } a + b + c = 0 \text{ then } a^3 + b^3 + c^3 = 3ab$$

$$22) a^2 + b^2 + c^2 - ab - bc - ca = 0 \Rightarrow a = b = c$$

$$23) (a)^{\sqrt[n]{a}} = (a\sqrt{a})^a \text{ then } a = \frac{9}{4}$$

24) If  $\frac{1}{a+b+c} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ , n is a positive odd integer then

$$\frac{1}{a^n} + \frac{1}{b^n} + \frac{1}{c^n} = \frac{1}{a^n + b^n + c^n}$$

25) If  $y = x^p \Leftrightarrow x = y^{vp}, y > 0 (p \neq o), x > o$

$$26) a^{p/q} = (a^p)^{\frac{1}{q}} = \sqrt[q]{a^p}, a \neq 0$$

$$27) (\sqrt[n]{a})^n = a$$

$$28) \sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$$

$$29) \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

$$30) \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$31) \sqrt[n]{a^n} = a, n \geq 0$$

$$32) a + \sqrt{b} = c + \sqrt{d} \Leftrightarrow a = c \text{ and } b = d$$

$$33) \sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y} \text{ then } \sqrt{a - \sqrt{b}} = \sqrt{x} - \sqrt{y}$$

$$34) \sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y} \text{ then } x = \frac{1}{2}(a + \sqrt{a^2 - b}) y = \frac{1}{2}(a - \sqrt{a^2 - b})$$

$$35) \sqrt{a + \sqrt{b} + \sqrt{c} + \sqrt{d}} = \frac{1}{2} \left[ \sqrt{\frac{bd}{c}} + \sqrt{\frac{bc}{d}} + \sqrt{\frac{cd}{b}} \right]$$

$$36) a + b = \left( a^{\frac{1}{3}} + b^{\frac{1}{3}} \right) \left( a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \right)$$

$$37) \quad a - b = \left( a^{\frac{1}{3}} - b^{\frac{1}{3}} \right) \left( a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \right)$$

$$38) (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

39) If  $a^x = b, b^y = c, c^z = a$  then  $xyz = 1$

$$40) x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$$

$$41) x^6 - y^6 = (x^3 - y^3)(x^3 + y^3)$$

$$43) \sqrt{x\sqrt{x\sqrt{x\ldots\ldots\ldots\infty}}} = x$$

$$44) \quad |x| = x, \text{if } x > 0 \quad = -x, \text{if } x < 0$$

$$45) |x| \leq a \text{ then } -a \leq x \leq a$$

46)  $|x| < a$  then  $-a < x < a$

47)  $|x| \geq a$  then  $x \geq a$  or  $x \leq -a$

48)  $|x| > a$  then  $x > a$  or  $x < -a$

$$49) |x|^2 = x^2$$

$$50) x_1y \varepsilon R \Rightarrow |x + y| \leq |x| + |y|$$

$$51) x_1y \varepsilon R \Rightarrow |x - y| \leq |x| - |y|$$

$$52) x_1y \varepsilon R \Rightarrow |x - y| \geq ||x| - |y||$$

$$53) x_1y \varepsilon R \Rightarrow |x + y| \geq ||x| - |y||$$

$$54) x_1y \varepsilon R \Rightarrow |xy| \geq |x||y|$$

$$55) x_1y \in R, y \neq 0 \Rightarrow \frac{x}{y} = \frac{|x|}{|y|}$$

$$56) |x - a| < l \Rightarrow a - l < x < a + l$$

$$57) \text{ INDETERMINATE FORMS:- } 1^\infty, 0^\theta, \infty^0, \infty - \infty, \frac{\infty}{\infty}, \frac{0}{0}, 0 \times \infty$$

$$58) \infty + \infty = \infty,$$

$$-\infty + -\infty = -\infty,$$

$$-\infty \times \infty = -\infty$$

$$(-\infty) \times (-\infty) = \infty$$

$$59) \text{ If } a, b > 0, a^b + b^a > 1$$

$$60) a > b \Rightarrow a + c > b + c$$

$$61) a > b \Rightarrow a - c > b - c$$

$$62) a < b \Rightarrow a + c > b + c$$

$$63) a < b \Rightarrow a - c > b - c$$

$$64) a < b \Rightarrow \frac{1}{a} > \frac{1}{b}, a \neq 0, b \neq 0$$

$$65) a < b \Rightarrow -a > -b$$

$$66) a > b \Rightarrow -a < -b$$

$$67) a^{x-y} \cdot a^{y-z} \cdot a^{z-x} = 1$$

$$68) a^{x(y-z)} \cdot a^{y(z-x)} \cdot a^{z(x-y)} = 1$$

69) LIMIT:- Let  $a, l \in \mathbb{R}$  and  $f$  is defined on  $(a - s, a) \cup (a, a + s)$  then  $l$  is said to be a limit of  $f$  at  $a$  if for each given  $\varepsilon > 0, \exists$

$s > 0, \exists x \in \mathbb{R}, 0 < |x - a| < s \Rightarrow |f(x) - l| < \varepsilon$  it is denoted by  $lt_{x \rightarrow a} f(x) = l$

$$70) \lim_{x \rightarrow a} x^n = a^n, n \in \mathbb{Z}^+$$

$$71) \lim_{x \rightarrow a} f(x) = f(a), f(x) \text{ is a polynomial.}$$

$$72) \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} (n \in \mathbb{R})$$

$$73) \lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}, m, n \in \mathbb{R}$$

$$74) \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 0, IF \deg[f(x)] < \deg[g(x)]$$

$$= \infty, IF \deg[f(x)] > \deg[g(x)]$$

$$75) \lim_{x \rightarrow a} \frac{a_0 x^n + a_1 x^{n-1} + \dots + a_n}{b_0 x^m + b_1 x^{m-1} + \dots + b_n} = \frac{a_0}{b_0} IF m = n$$

$$= 0 IF m < n$$

$$= \infty IF m < n$$

$$76) \lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$77) \lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$78) \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$79) \lim_{x \rightarrow a} k f(x) = k \lim_{x \rightarrow a} f(x)$$

$$80) \lim_{x \rightarrow \infty} [\sqrt{x^2 + ax + b} - x] = \frac{a}{2}$$

$$81) \lim_{x \rightarrow 0} (1 + ax)^{\frac{1}{x}} = e^a$$

$$82) \lim_{x \rightarrow 0} \frac{a^{x+t_1+b}}{a^{x+t_2+c}} = a^{t_1-t_2}$$

$$83) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$84) \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$85) \left( \frac{lt}{x} \rightarrow 0 \right) \frac{\sin a_x}{x} = a$$

$$86) \left( \frac{lt}{x} \right) \frac{\tan a_x}{x} = a$$

$$87) \left( \frac{lt}{x \rightarrow \rho} \right) \frac{x^n}{\sqrt{1+x^n} - \sqrt{1-x^n}} =$$

$$88) \left( \lim_{x \rightarrow 0} \right) \frac{\sqrt[n]{1+x} - \sqrt[n]{1-x}}{x} = \frac{2}{n}$$

$$89) \left( \frac{lt}{x} \right) \frac{\sqrt[n]{a+x} - \sqrt[n]{a-x}}{x} = \frac{2}{n}(a)$$

$$90) \left( \frac{lt}{x^n} \right)^{\frac{\sqrt{2+x^n}-\sqrt{2-x^n}}{x^n}} = \frac{1}{\sqrt{2}}$$

$$91) \left( \begin{matrix} lt \\ x \rightarrow \rho \end{matrix} \right) \frac{\sqrt{a+x} - \sqrt{a}}{x} = \frac{1}{\frac{x}{\sqrt{a}}} = \left( \begin{matrix} lt \\ x \rightarrow \rho \end{matrix} \right) \frac{\sqrt{a} - \sqrt{a-x}}{x}$$

$$92) \left( \frac{lt}{x} \right) \frac{a^x - 1}{x} = log a$$

$$93) \sqrt{x + \sqrt{x + \sqrt{x + \dots}}} = \frac{1 \pm \sqrt{1+4x}}{2}$$

$$94) \text{ If } \left( \begin{matrix} lt \\ x \rightarrow a \end{matrix} \right) f(x) = l, \left( \begin{matrix} lt \\ y \rightarrow l \end{matrix} \right) g(y) = m \text{ then } \left( \begin{matrix} lt \\ x \rightarrow a \end{matrix} \right) gof(x) = m$$

## (6) PROGRESSIONS

1)  $a, a + d, a + 2d \dots \dots \dots \dots \dots$  Form an AP

$$2) t_n = a + (n - 1)d$$

$$3) \text{ In A.P , } s_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [a + l]$$

4) If  $a, b, c$  are in A.P then  $2b = a + c$

5) The arithmetic mean (A.M) of  $a, b$  is  $\frac{a+b}{2}$

6) If there are 'n' A.M's between  $a$  and  $b$  then  $d = \frac{b-a}{n+1}$

7) If  $t_1 = a, t_n = l, s$  is the sum of  $n$  terms in A.P then  $d = \frac{l^2 - a^2}{2s - (l+a)}$

8) If  $t_p = a, t_q = b$  in an A.P then  $t_p + q = \frac{p+q}{2} [a + b + \frac{a-b}{p-q}]$

9) If  $t_1 = a, t_2 = b, t_n = c$  in an A.P then  $s_n = \frac{(a+c)(b+c-2a)}{2(b-a)}$

10)  $\Sigma n = \frac{n(n+1)}{2}$

11)  $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

12)  $\Sigma n^3 = \frac{n^2(n+1)^2}{4}$

13)  $\Sigma n^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$

14)  $\Sigma n^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$

15) If  $a, b, c$  are in A.P then

1)  $a+k, b+k, c+k$  are also in AP

2)  $a-k, b-k, c-k$  are also AP  $K \in R$

16) In an A.P ,  $d = t_2 - t_1$

$$= t_3 - t_2$$

$$= t_n - t_{n-1}$$

17) In an A.P if  $s_n$  is given then

$$t_1 = s_1$$

$$t_2 = s_2 - s_1$$

$$t_3 = s_3 - s_2$$

18) If 'n' A.M's are inserted between a and b than the sum of means  $= \frac{n}{2}(a + b)$

19) If a,b,c are in AP then

1)  $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \dots \dots \dots$  are also in AP, KER

2)  $ak, bk, ck$  are also in AP , KER

20)  $a, ar, ar^2, \dots \dots \dots$  form a G.P

21) In GP,  $t_1 = a, t_n = ar^{n-1}$

22) In GP ,  $s_n = \frac{a(r^n - 1)}{r - 1}, r > 1$

$$= \frac{a(1 - r^n)}{1 - r}, r < 1$$

23) In G.P ,  $s_n = \frac{l_r - a}{r - 1}, (r \neq 1)$

24) In G.P ,  $s_\infty = \frac{a}{1 - r}$

25) G.M of  $a, b$  is  $\sqrt{ab}$

26) If  $a, b, c$  are in G.P then  $b^2 = ac$

27) If there are 'n' GM's between  $a$  and  $b$  then  $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

- 28) If  $a, b, c$  are in G.P then  $a^k, b^k, c^k$  are also in AP
- 29) The reciprocals of all terms in G.P are also in G.P
- 30) In an G.P if each term is multiplied or divided by a fixed number the resulting series is also in G.P
- 31) If 3 numbers are in G.P then we can take as  $\frac{a}{r}, a, ar$
- 32) If 4 numbers are in G.P then we can take as  $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
- 33) In G.P,  $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}$
- 34) If  $k^a, k^b, k^c$ , are in G.P  $a, b, c$  are in A.P
- 35) If  $a, b, c$  are in G.P then  $\log_a, \log_b, \log_c$  are in A.P
- 36) If 'n' G.M's are inserted between  $a, b$ , then product of 'n' G.M's is  $(ab)^{\frac{n}{2}}$
- 37) If m,n are A.M , G.M between two numbers then the numbers are  $m \pm \sqrt{m^2 - n^2}$
- 38) If A,G are A.M and G.M of the roots of a quadratic equation then the quadratic equation is  $x^2 - 2Ax + G^2 = 0$
- 39) If  $a, a+d, a+2d, \dots$  are in A.P then  $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots$  are in harmonic progression
- 40) In H.P,  $t_1 = \frac{1}{a}, t_n = \frac{1}{a+(n-1)d}$
- 41) If  $a, b, c$  are in H.P then  $b = \frac{2ac}{a+c}$
- 42) H.M of  $a, b$  is  $\frac{2ab}{a+b}$
- 43) If  $x_1, x_2, \dots, x_n$  are 'n' H.M's between  $a, b$ , then  $x_n = \frac{ab(n+1)}{b(n+1)n(a-b)}$
- 44) If H is a H.M between a and b then  $\frac{1}{H-a} + \frac{1}{H-b} = \frac{1}{a} + \frac{1}{b}$
- 45) If  $H_1, H_2, \dots, H_n$  are 'n' H.M's between a and b then  $\frac{H_1+a}{H_1-a} + \frac{H_n+b}{H_n-b} = 2n$
- 46) (A.M)  $\times$  (H.M) = (G.M)<sup>2</sup>
- 47) (A.M)  $\geq$  G.M  $\geq$  HM
- 48) If  $p = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$   
 If  $n=0$ , P is A.M of a,b  
 If  $n=-1/2$ , P is G.M of a,b  
 If  $n=-1$ , P is H.M of a,b
- 49) If 'n' A.M's are inserted between a,b then  $r^{th}$  A.M =  $a + \frac{r(b-a)}{n+1}$

50) If 'n' G.M's are inserted between a and b then  $r^{th}$  G.M =  $a\left(\frac{b}{a}\right)^{\frac{r}{n+1}}$

51) If 'n' G.M's are inserted between a and b then  $r^{th}$  Hm =  $\frac{(n+1)ab}{b(n+1)+r(a-b)}$

52) If  $a_1, a_2, a_3, \dots, a_n$  are in H.P then

$$\frac{a_1}{a_2 + a_3 + \dots + a_n}, \frac{a_2}{a_1 + a_3 + a_4 + \dots + a_n}, \frac{a_n}{a_1 + a_2 + \dots + a_{n-1}}$$

53) The series of the form  $a \cdot 1 + (a+d)r + (a+2d)r^2 + \dots + [a + (n-1)d]r^{n-1}$

Is an Arthemetic - geometric series  $s_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$

$$s_\infty = \frac{a}{1-r} + \frac{d_r}{(1-r)^2}$$

54) Sum of first 'n' odd numbers =  $1+3+5+\dots+n$  terms =  $n^2$

55) Sum of first 'n' even numbers =  $2+4+6+\dots=n(n+1)$

56) In an A.P,  $t_m = n, t_n = m$  then  $t_{m+n} = 0$

57) If  $a_1, a_2, \dots, a_n$  are in AP then  $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_n a_1} = \frac{n-1}{a_1 a_n}$

58) If  $a, b, c$  are in A.P then

1)  $a^2(b+c), b^2(c+a), c^2(a+b)$  are also in AP

2)  $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$  are also in AP

59) In an A.P,  $t_p = a_1, t_q = b_1, T_r = c$  then  $a(q-r) + b(r-p) + c(p-q) = 0$

60) In a G.P,  $t_{p+q} = m, t_{p-q} = n$  then  $t_p = \sqrt{mn}$

61) In a H.P,  $t_m = n, t_n = m$  then  $t_{m+n} = \frac{mn}{m+n}$

62) If  $a, b, c$  are in AP,  $a^2, b^2, c^2$  are in HP then  $a = b = c$

63) If  $a, b, c$  are in GP and  $a^{1/x}, b^{1/y}, c^{1/z}$  then  $x, y, z$  are in AP

64)  $1.2.3 + 2.3.4 + 3.4.5 + \dots$  n terms =  $\frac{n(n+1)(n+2)(n+3)}{4}$

65)  $1 + (1+3) + (1+3+5) + \dots = \frac{n(n+1)(2n+1)}{6}$

66)  $(1^2) + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) = \frac{n(n+1)^2(n+2)}{1^2}$

67)  $(En^3)(En) = (En^2)^2$  if  $n=1$

68)  $1 + (2 + 3 + 4) + (5 + 6 + 7 + 8 + 9) + \dots = (n-1)^3 + n^3$

69)

$1 + (2+3) + (4+5+6+7) + (8+9+10+11+12+13+14+15) + \dots$  n terms =  $2^{n-2}[2^n + 2^{n-1} - 1]$

70)  $1.2 + 2.3 + 3.4 + \dots \dots \dots n \text{ terms} = \frac{n(n+1)(n+2)}{3}$

71) If  $a, b$  are two numbers then  $a + b \geq 2\sqrt{ab}$

72) If  $a, b, c$  are 3 numbers then  $a + b + c \geq 3\sqrt[3]{abc}$

73) If  $a, b, c, d$  are 4 numbers then  $a + b + c + d \geq 4\sqrt[4]{abcd}$

74) If  $a_1, a_2, \dots, a_n$  are 'n' numbers then  $a_1 + a_2 + \dots + a_n \geq \sqrt[n]{a_1 a_2 \dots a_n}$

75) Cauchy – Schwarz inequality If  $a_1, a_2, a_3, \dots, a_n$  and  $b_1, b_2, \dots, b_n$  be two sets of real numbers then

$$(a_1^2 + a_2^2 + \dots + a_n^2)(b_1^2 + b_2^2 + \dots + b_n^2) \geq (a_1 b_1 + a_2 b_2 + \dots + a_n b_n)$$

76) Weierstrass inequality

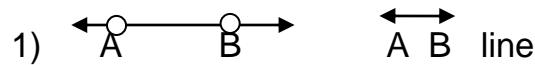
1) If  $a_1, a_2, \dots, a_n$  be all +ve quantities and  $a_1 + a_2 + \dots + a_n = s_n$  then  $(1 + a_1)(1 + a_2) \dots (1 + a_n) > 1 + s_n$

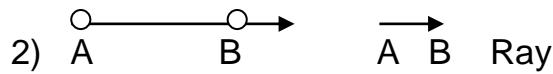
2)  $a_1, a_2, \dots, a_n$  are +ve and less than 1 then  $(1 - a_1)(1 - a_2) \dots (1 - a_n) > 1 - s_n$

77) 1)  $\left( \frac{a_1^m + a_2^m + \dots + a_n^m}{n} \right) \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)^m$  if  $m \geq 1$

2)  $\left( \frac{a_1^m + a_2^m + \dots + a_n^m}{n} \right) \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)^m$ ,  $0 < m < 1$

## (7) GEOMETRY



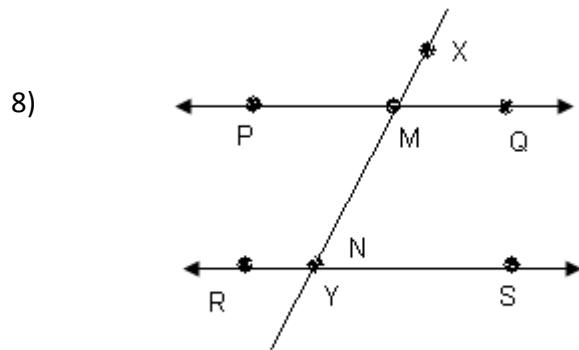


4) If  $n$  points are given then no. of lines formed =  $\frac{n(n-1)}{2}$

5) For  $n$  sides of a polygon no. of diagonals formed =  $\frac{n(n-3)}{2}$

6) If  $\angle A + \angle B = 90^\circ$  then are complementary angles

7) If  $\angle A + \angle B = 180^\circ$  then  $LA, LB$  are supplementary angles



a)  $\angle PMN = \angle MNS$

$$\angle QMN = \angle MNR$$

b)  $\angle XMQ = \angle MNS$

$$\angle QMN = \angle SNY$$

$$\angle XMP = \angle MNR$$

$$\angle PMN = \angle RNY$$

c)  $\angle QMN + \angle MNS = 180^\circ$

$$\angle PMN + \angle MNR = 180^\circ$$

d)  $\angle XMQ + \angle SNY = 180^\circ$

$$\angle XMP + \angle LRNY = 180^\circ$$

9) Sum of three angles in a triangle  $= 180^\circ$  i.e.:  $L A + L B + L C = 180^\circ$

10) In any triangle sum of two sides is always greater than third side.

11) In any triangle difference of two sides is always less than third side.

12) The exterior angle of a triangle is equal to the sum of two opposite interior angles.

13) In a right angle triangle

$$AB^2 + BC^2 = AC^2 \quad \text{if } L B = 90^\circ$$

$$AB^2 + CA^2 = BC^2 \quad \text{if } L A = 90^\circ$$

$$BC^2 + CA^2 = AB^2 \quad \text{if } L C = 90^\circ$$

14) In a obtuse angle triangle ( $AD \perp BC$ )

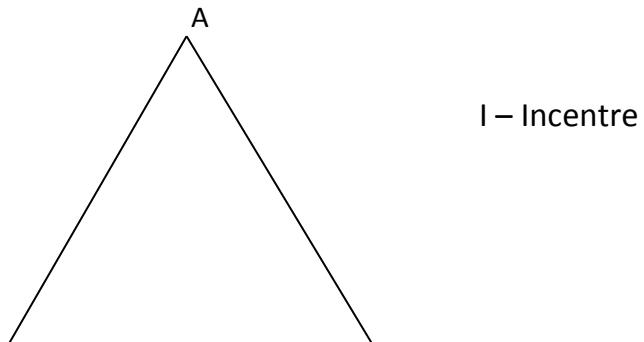
$$AC^2 = AB^2 + BC^2 + 2BC \cdot BD$$

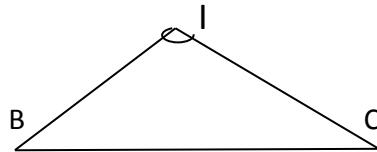
15) In a acute angle triangle ( $AD \perp BC$ )

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$$

16) In a right angle triangle circumcentre mid point of hypotenuse.

17)





$$\angle BIC = 90^\circ + \frac{\angle A}{2}$$

$$\angle AIB = 90^\circ + \frac{\angle C}{2}$$

$$\angle LCI = 90^\circ + \frac{\angle A}{2}$$

- |                           |                      |
|---------------------------|----------------------|
| 18) Point of intersection | centre               |
| Altitudes                 | Orthocenter (O or H) |
| Medians                   | Centroid (G)         |
| Perpendicular bisections  | Circum centre(S)     |
| Internal angle bisectors  | In centre (I)        |
- 19) If the angular bisectors of angle A meet BC at D, then  $\frac{BD}{DC} = \frac{AB}{AC}$
- 20) In a right angle triangle, the vertex where the right angle is formed is the ortho centre
- 21) In ABC, G is centroid D,E,F are mid points of BC, CA, AB, then

$$AG:GD = 2:1$$

$$BG:GE = 2:1$$

$$CG:GF = 2:1$$

22) **APOLLONIUS THEOREM**:- In ABC, if AD is the median from A to side BC meeting BC at its midpoint D, then  $= 2(AD^2 + BD^2) = AB^2 + AC^2$

23) In an equilateral triangle O,G,S,I are coincide

24) In an isosceles triangle O,G,S,I are collinear

25) In a right angle triangle, the length of the median drawn to the hypotenuse is equal to half the hypotenuse

26) In a triangle if two medians are equal then it given isosceles triangle

27)



AD,BE,CF are Medians

$$4(AD+BE+CF) > 3(AB+BC+CA)$$

28) If O,G,S of a triangle are collinear then G divides OS in the ratio 2:1

$$OG:GS = 2:1$$

29) If O is the ortho centre of ABC then,

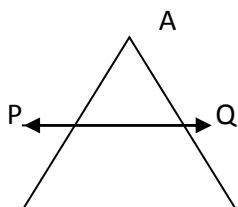
A is the orthocenter of OBC

B is the orthocenter of OCA

C is the orthocenter of OAB

30) If G is the centroid ABC then AG:GD=2:1

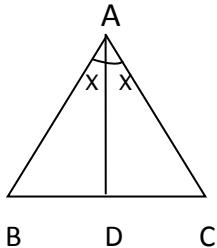
31)



B C

If  $\overleftrightarrow{PQ} \parallel \overline{BC}$  Then  $\frac{AP}{PB} = \frac{AQ}{QC}$

32)



If  $\overleftrightarrow{PQ} \parallel \overline{BC}$  Then  $\frac{AP}{PB} = \frac{AQ}{QC}$

33) In two similar triangles

1) Ratio of sides = Ratio of heights=Ratio of medians

=Ratio of angular bisectors

=Ratio of in radii

2) Ratio areas = Ratio of squares of corresponding sides

34) If ABC is an equilateral triangle

$$\text{Area} = \frac{\sqrt{3}}{4} a^2, \text{ Height} = \frac{\sqrt{3}}{2} a$$

35) Area of a triangle =  $\frac{1}{2}$  base X height

36) Sum of 4 angles in a quadrilateral =360

37) In a cyclic quadrilateral

$$\text{LA+LC} = 180^0, \text{ LB+LD} = 180^0$$

38) Area of a quadrilateral =  $\frac{1}{2} d(H_1 + H_2)$

39) Side of a Rhombus =  $\frac{1}{2} \sqrt{d_1^2 + d_2^2}$

40) Area of Rhombus =  $\frac{1}{2} d_1 d_2$

41) Sum of interior angles of a convex polygon

$$= (2n-4) \times 90^0$$

42) Each interior angle =  $\frac{(2n-4) \times 90^0}{n}$

43) Sum of exterior angles of a polygon =  $360^0$

44) Each exterior angle of A POLYGON =  $\frac{360^0}{n}$

45) Area of a regular polygon =  $\frac{1}{2} \times (\text{Perimeter}) \times (\text{Perpendicular distance from the centre to any side})$

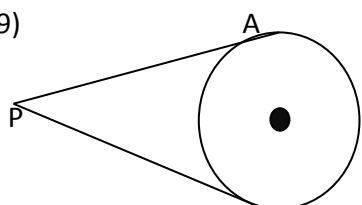
46) Length of tangent =  $\sqrt{d^2 - r^2}$

47) Length of direct common tangent =  $\sqrt{d^2 - (r_1 - r_2)^2}$

48) Length of transversal common tangent

$$= \sqrt{d^2 - (r_1 + r_2)^2}$$

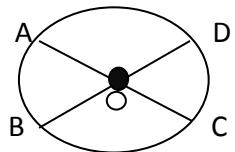
49)



B

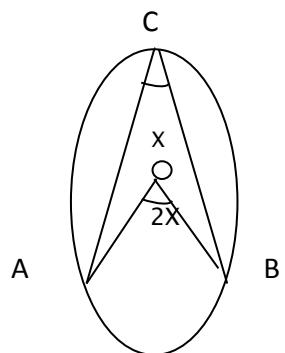
$$PA = PB$$

50)

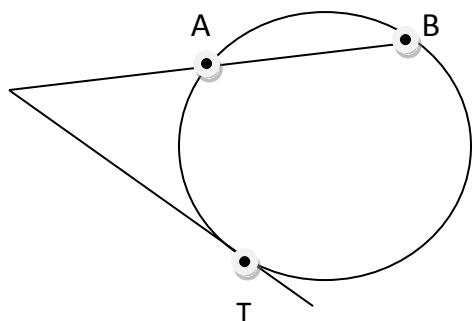


$$OA \times OC = OB \times OD$$

51)

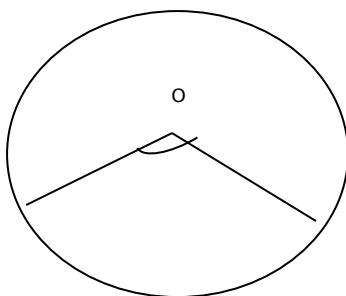


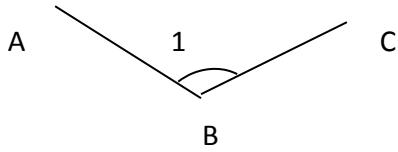
53)



$$PA \times PB = PT^2$$

54)



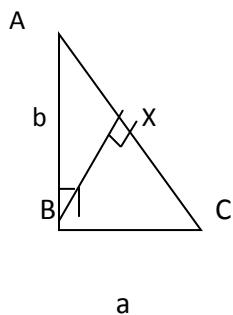


55) In a triangle ABC, AD is drawn perpendicular to BC, then

$$AB^2 - BD^2 = AC^2 - CD^2$$

56) ABCD is a parallelogram. Then  $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

57) Altitude  $BX = \frac{ab}{\sqrt{a^2+b^2}}$



58) The radii of two circles are R and r

The distance between their centers is 'd'

Then i)  $d > R + r \leftrightarrow 4$  Tangents

ii)  $d = R + r \rightarrow 3$  Tangents

iii)  $d < R + r \rightarrow 2$  Tangents

iv)  $d = |R-r| \rightarrow 1$  Tangent

v)  $d < R - r \rightarrow 0$

59) If the angles of a triangle are in the ratio 1:1:1, the corresponding sides are 1:1:1.

60) If the angles of a triangle are in the ratio 1:1:2 then the corresponding sides are 1:1: $\sqrt{2}$

61) If the angles in a triangle are in the ratio 1:2:3, then the corresponding sides are in the ratio  $1:\sqrt{3}:2$

62) In  $\Delta ABC$ , if AD, BE, CF are medians and  $AB = C$ ,  $BC = a$ ,  $CA = b$  then

$$AD = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$$

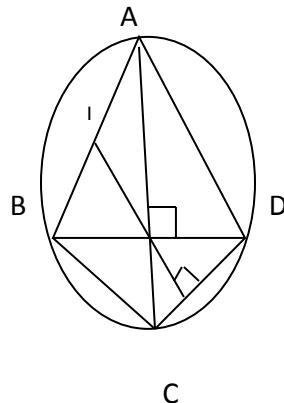
$$BE = \frac{1}{2} \sqrt{2c^2 + 2a^2 - b^2}$$

$$CF = \frac{1}{2} \sqrt{2a^2 + 2b^2 - c^2}$$

63) The sum of the squared of the medians of a triangle  $= \frac{3}{4}(a^2 + b^2 + c^2)$

64) P is a point on the circumcircle  $\square ABC$ , PI, PM, PN are drawn perpendicular to  $\overleftrightarrow{BC}$ ,  $\overleftrightarrow{AC}$ ,  $\overleftrightarrow{BA}$  respectively then  $\overleftrightarrow{LMN}$  are collinear LMN is called the simon's rule  $\overleftrightarrow{LMN}$  is also called the pedal line.

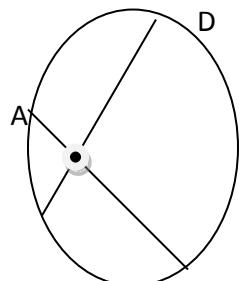
65)



C

If  $\angle AED = 90^\circ$ ,  $\angle EGD = 90^\circ$  then  $AI = IB$

66)

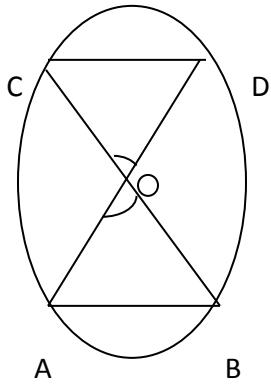


C P

B

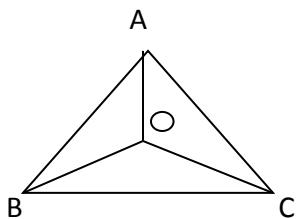
If  $AB = CD$  then  $PA = PC$  and  $PB = PD$

67)



If  $AB = CD$  then  $Lx = Ly$

68)



'o' is any point in the interior of  $\Delta ABC$  then  $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

69) Centroid 'G' of tetrahedron ABCD divides the line joining any vertex to centroid of its opposite face in 3:1 ratio

70) Internal angles bisector of angle A of  $\Delta ABC$  divides the opposite BC in the ration AB:AC

71) If  $a=Bc$ ,  $b=CA$ ,  $c=Ab$  of ABC And if I is in centre then I divides the internal angular bisector of angle A in the ratio  $AI : ID = b+c : a$

72) In ABC if  $AB^2 + BC^2 = AC^2 \rightarrow \text{right triangle}$

$AB^2 + BC^2 > AC^2 \rightarrow \text{Acutes angle triangle}$

$$AB^2 + BC^2 < AC^2 \rightarrow \text{Obtuse angle triangle}$$

73) If D,E,F be the midpoint of BC,CA,AB respectively of  $\Delta ABC$  then

- i) Centroid of  $\Delta ABC$  = Centroid of  $\Delta DEF$
- ii) Circumcentre of  $\Delta ABC$  = Ortho centre of  $\Delta PQR$

74)  $\Delta ABC$ , ON: NG: GS = 3:1:2 here S-circumcentre, G-centroid, O-orthocentre, N-nine point circle

75) If sides of cyclic quadrilateral are a,b,c,d and x,y are diagonals then  $x =$

$$\sqrt{\frac{(ad+bc)}{(ab+cd)}}(ac + bd)$$

$$\text{and } y = \sqrt{\frac{ab+cd}{ad+bc}}(ac + bd)$$

$$\text{and } xy = ac + bd$$

## 8) ANALYTICAL GEOMETRY

1) Equation of x axis is  $y=0$

2) Equation of y axis is  $x=0$

3) Equation of any line parallel to x axis is  $y=k$

4) Equation of any line parallel to y axis is  $x=k$

5) Distance from  $p(x_1, y_1)$  to x axis is  $|y_1|$

6) Distance from  $p(x_1, y_1)$  to y axis is  $|x_1|$

7) Slope of x axis = 0

8) Slope of y axis is not defined

9) Slope (m) =  $\frac{y_2 - y_1}{x_2 - x_1}$

10) If  $\theta$  be inclination made by a line with positive x axis then slope (m) =  $\tan \theta$

11) The distance between  $A(x_1, y_1), B(x_2, y_2)$  is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

12) Midpoint of  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

13) the distance from origin to  $(x_1, y_1)$   $\sqrt{x_1^2 + y_1^2}$

14) The point which divides the line joining points  $A(x_1, y_1), B(x_2, y_2)$

in the ratio m:n internally is  $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

15) The point which divides the line joining points  $A(x_1, y_1), B(x_2, y_2)$  in the ratio

m:n externally is  $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

16) The centroid  $\Delta ABC, A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$  is  $G =$

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

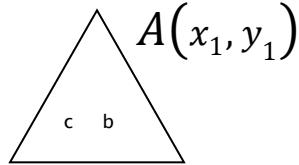
17) The point which divides  $\sim$  line segment the ratio 2:1 or 1:2 is called a point of trisection

18) The ratio in which x axis divides the line segment joining  $(x_1, y_1), (x_2, y_2)$  is  $-y_1:y_2$

19) The ratio in which y axis divides the line segment joining  $(x_1, y_1), (x_2, y_2)$  is  $-x_1:x_2$

20) The ratio in which  $(x, y)$  divides the line segment joining  $(x_1, y_1), (x_2, y_2)$  is  $x_1 - x:x - x_2$

21)



$$B(x_2, y_2) \quad C(x_3, y_3)$$

$$\text{In centre } I = \left( \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

$$22) \text{Excentre opposite to A is } I_1 = \left( \frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c} \right)$$

$$23) \text{Excentre opposite to B is } I_2 = \left( \frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c} \right)$$

$$24) \text{Excentre opposite to 'C' is } I_3 = \left( \frac{ax_1 - bx_2 - cx_3}{a+b-c}, \frac{ay_1 - by_2 - cy_3}{a+b-c} \right)$$

25) If A,B,C are collinear then

$$1) \text{Area of } \Delta ABC = 0$$

$$2) Ab+BC=Ac \text{ (or)} \ BA+CA=Bc \text{ (or)} \ Bc+AC=Ab$$

26) If  $D(x_1, y_1), E(x_2, y_2), F(x_3, y_3)$  are the mid points of the sides

$$\overline{BC}, \overline{CA}, \overline{AB}, \text{ of } \Delta ABC, \text{ then } A = (x_2 + x_3 - x_1, y_2 + y_3 - y_1)$$

$$B = (x_3 + x_1 - x_2, y_3 + y_1 - y_2)$$

$$C = (x_1 + x_2 - x_3, y_1 + y_2 - y_3)$$

27) If  $A(x_1, y_1) B(x_2, y_2) C(x_3, y_3)$  are three consecutive vertices of a parallelogram, then the fourth vertex =  $(x_1 + x_2 - x_3, y_1 + y_3 - y_2)$

28) If a point P divides the line segment joining the points A,B in the ratio l:m then the point Q which divides  $\overline{AB}$  in the ratio l:-m is called harmonic conjugate of P with respect to A and B

29) If 'S' is the circum centre of  $\Delta ABC$  then  $SA=SB=SC$

30) If  $A(x_1, y_1)$   $B(x_2, y_2)$   $C(x_3, y_3)$  Of AABC orthocenter =  $(x_1, y_2)$

31) If  $A(x_1, y_1)$   $B(x_2, y_2)$   $C(x_3, y_3)$  vertices of  $\Delta ABC$  then

$$\text{cicumcentre} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

32) If  $A(x_1, y_1)$   $B(x_2, y_2)$   $C(x_3, y_3)$  are vertices of  $\Delta ABC$  then

$$\begin{aligned} \text{Area of AABC} &= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} x_1 - & x_2 - & x_1 - & x_3 \\ y_1 - & y_2 - & y_1 - & y_3 \end{vmatrix} \\ &= \frac{1}{2} |\Sigma(x_1y_2 - x_2y_1)| \end{aligned}$$

33) The area of the quadrilateral with consecutive

$$\begin{aligned} \text{vertices } A(x_1, y_1) B(x_2, y_2) C(x_3, y_3) D(x_4, y_4) \text{ is} &= 1/2 \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \\ &= \frac{1}{2} \begin{vmatrix} x_1 - & x_3 & x_2 - & x_4 \\ y_1 - & y_3 & y_1 - & y_4 \end{vmatrix} \end{aligned}$$

34) If  $A(x_1, y_1)$   $B(x_2, y_2)$  are two vertices of an equilateral triangle , then its third vertex  $\left( \frac{x_1+x_2 \pm \sqrt{3}(y_1-y_2)}{2}, \frac{y_1+y_2 \mp (x_1-x_2)}{2} \right)$

35) If  $(x_1, y_1)$   $(x_2, y_2)$  are two vertices of a triangle whose centroid is  $(x, y)$  then third vertex is  $(3x - x_1 - x_2, 3y - y_1 - y_2)$

36) If  $A(x_1, y_1)$  ;  $B(x_2, y_2)$  end points of the hypotenuse of a right angled isosceles triangle, then its third vertex =  $\left[ \frac{x_1+x_2 \pm (y_1-y_2)}{2}, \frac{y_1+y_2 \mp (x_1-x_2)}{2} \right]$

37) Area of triangle formed by  $o(o, o)$   $A(x_1, y_1)$ ,  $B(x_2, y_2)$  is  $1/2|x_1y_2 - x_2y_1|$

38) Area of triangle formed by  $(0,0)$ ,  $A(a,0)$ ,  $B(0,b)$  is  $\frac{1}{2}|ab|$

39) If one end point of diameter of a circle is  $(a,b)$  and centre  $(0,0)$  then the other end  $(-a,-b)$

40) The line  $y = mx$  passing through origin

41) The line  $y = mx + c$  passing through  $(0,c)$

42) Slope of  $ax + by + c = 0$  is  $-\frac{a}{b}$

43) Slope of  $y = mx + c$  is  $m$

44) Slope of  $\frac{x}{a} + \frac{y}{b} = 1$  is  $-\frac{b}{a}$

45) For the line  $ax + by + c = 0$

$$X - \text{intercept} = -\frac{c}{a}$$

$$Y - \text{intercept} = -\frac{c}{b}$$

46) For the line  $\frac{x}{a} + \frac{y}{b} = 1$

$$X - \text{intercept} = a$$

$$Y - \text{intercept} = b$$

47) Slope =  $m$ ,  $y$  intercept =  $C$  then equation of line is of  $y = mx + c$

48) Slope =  $m$ ,  $x$  intercept =  $C$  then equation of line is of  $y = mx(x - c)$

49) Eq of line passing through the point with slope is  $y - y_1 = m(x - x_1)$

50) Eq of line passing through two points

$$(x_1, y_1), (x_2, y_2) \text{ is } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

51) If  $a_1x + b_1y = c_1$  and  $a_2x + b_2y = c_2$  are any two different equations and  $a_1b_2 \neq a_2b_1$  then  $x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$

$$y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

52) Area formed by the line  $ax + by + c = 0$  with coordinate axis is  $\frac{c^2}{2|ab|}$

53) The area of the triangle formed by the line  $\frac{x}{a} + \frac{y}{b} = 1$  with coordinate axes is  $\frac{1}{2}|ab|$

54) The equation of the horizontal line passing through  $(x_1, y_1)$  is  $y = y_1$

55) The equation of the vertical line passing through  $(x_1, y_1)$  is  $x = x_1$

56) The equation of the line parallel to  $ax + by + c = 0$  and passing through  $(x_1, y_1)$  is  $a(x - x_1) + b(y - y_1) = 0$

57) The equation of the line perpendicular to  $ax + by + c = 0$  and passing through is  $b(x - x_1) - a(y - y_1) = 0$

58) The perpendicular distance from to  $ax + by + c = 0$  is  $\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

59) The distance between two parallel lines  $ax + by + c = 0$  and  $ax + by + c' = 0$  is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

60) The ratio in which the  $ax + by + c = 0$  divides the line segment joining  $A(x_1, y_1), B(x_2, y_2)$  is  $(ax_1 + by_1 + c):(ax_2 + by_2 + c)$

61) If  $A(x_1, y_1), B(x_2, y_2)$  are two points and  $ax + by + c = 0$  is a line

(i) if  $ax_1 + by_1 + c$  and  $ax_2 + by_2 + C$  have same sign then two points lie on same side of the line

(ii) If  $ax_1 + by_1 + c$  and  $ax_2 + by_2 + C$  have opposite sign then they lies on opposite sides of the line.

62) If  $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0, a_3x + b_3y + c_3 = 0$ , to be concurrent if  $a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$

63) ' $\theta$ ' is angle between the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  then Tan

$$\theta = \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}$$

64) The foot of the perpendicular to  $(x_1, y_1)$  to the x axis is  $(x_1, 0)$

65) The foot of the perpendicular to  $(x_1, y_1)$  to y axis is  $(0, y_1)$

66) The foot of perpendicular from  $(x_1, y_1)$  to the line  $y = k$  is  $(x_1, k)$ .

67) The foot of the perpendicular from  $(x_1, y_1)$  to the line  $x = k$  is  $(k, y_1)$

68) If  $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$  are two parallel lines then  $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

69) If  $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$  two perpendicular lines then  $a_1a_2 + b_1b_2 = 0$

70) The image of  $(x_1, y_1)$  with respect to the x axis is  $(x_1, -y_1)$

71) The image of  $(x_1, y_1)$  with respect to the y axis is  $(-x_1, y_1)$

72) The area of the triangle formed by  $y = m_1x + c_1, y = m_2x + c_2, y = m_3x + c_3$  is  $\frac{1}{2} \left| \frac{(c_1 - c_2)^2}{m_1 - m_2} + \frac{(c_2 - c_3)^2}{m_2 - m_3} + \frac{(c_3 - c_1)^2}{m_3 - m_1} \right|$

73) The area of the parallelogram formed by  $a_1x + b_1y + c_1 = 0, a_1x + b_1y + c_2 = 0, a_2x + b_2y + c_2 = 0$  is  $\left| \frac{(c_1 - c_2)(d_1 - d_2)}{a_1b_2 - a_2b_1} \right|$

74) The ortho centre of the triangle formed by  $(0, 0)$ ,  $(x_1, y_1)$   $(x_2, y_2)$  is  $[-(y_1 - y_2)K, K(x_1 - x_2)]$  where  $K = \frac{x_1 x_2 + y_1 y_2}{x_1 y_2 - x_2 y_1}$

75) Area of triangle formed by the lines  $y = mx$ ,  $y = m_1 x$ ,  $y = c$  is  $\frac{c^2(m_1 - m)}{2m_1 m}$

76) Area of the triangle formed by the lines  $y = ax - bc$ ,  $y = bx - ca$ ,  $y = cx - ab$  is  $\frac{(a-b)(b-c)(c-a)}{2}$

77) If the straight line joining the points  $(a,b)$   $(c,d)$  subtends an angle  $\theta$  at the origin then  

$$\cos\theta = \frac{ac+bd}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}$$

78) The area of the triangle formed by  $\left(a, \frac{1}{a}\right)$ ,  $\left(b, \frac{1}{b}\right)$ ,  $\left(c, \frac{1}{c}\right)$  is  $\left| \frac{(a-b)(b-c)(c-a)}{2abc} \right|$

79) The equation to the perpendicular bisector of the line segment joining  $(x_1, y_1)$   $(x_2, y_2)$  is  $x(x_1 - x_2) + y(y_1 - y_2) = \frac{(x_1^2 + y_1^2) - (x_2^2 + y_2^2)}{2}$

80) Equation of the line passing through  $(x_1, y_1)$  and making an angle  $\alpha$  with the line  $y = mx + c$  is  $y - y_1 = \tan(\theta \pm \alpha)(x - x_1)$  where  $m = \tan\theta$

81) If two lines having slopes  $m_1, m_2$  are

Parallel  $\rightarrow m_1 = m_2$

Perpendicular  $\rightarrow m_1 m_2 = -1$

82) Any point on x axis can be taken as  $(x, 0)$  and on y axis is  $(0, y)$

83) Area of triangle =  $\frac{\text{(in radius)} \times \text{(perimeter)}}{2}$

84)  $\frac{x}{a} + \frac{y}{b} = 1$  Passing through  $(a, 0)$  and  $(0, b)$

85)  $ax + by + c = 0$  Passing through  $\left(-\frac{c}{a}, 0\right)$ ,  $\left(0, -\frac{c}{a}\right)$

86)  $y = mx + c$  Passing through (0,c) and  $\left(-\frac{c}{m}, 0\right)$

87)  $x = my + c$  Passing through (c,0) and  $\left(0, -\frac{c}{m}\right)$

## (9) TRIGONOMETRY

1) 1 complete angle  $= 360^\circ = 400^g = 2\pi^c$

2) 1 Straight angle  $= 180^\circ = 200^g = \pi^c$

3) 1 Right angle  $= 90^\circ = 100^g = \frac{\pi^c}{2}$

4)  $\frac{D}{90} = \frac{G}{100} = \frac{C}{\pi/2}$

5)  $1^\circ = 0.01746$  radians

6)  $1^c = 57.16^1$

7) Area of sector  $= \frac{1}{2} lr$

$$= \frac{1}{2} r^2 \theta$$

8) Length of sector  $= \frac{x}{360^\circ} \times 2\pi r$

$$= r\theta$$

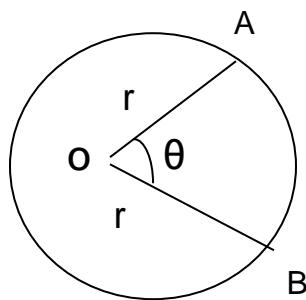
9) Area of minor segment

$$= \frac{1}{2r^2(\theta - \sin\theta)}$$

Area of major segment

$$= \frac{1}{2} r^2 [\pi - 1/2(\theta - \sin\theta)]$$

10)  $\sin\theta = \frac{\text{opposite side to } \theta}{\text{hypotenuse}}$



$$11) \cos\theta = \frac{\text{Adjacent side to } \theta}{\text{Hypotenuse}}$$

$$12) \tan\theta = \frac{\text{opposite side to } \theta}{\text{Adjacent side to } \theta}$$

$$13) \cot\theta = \frac{\text{adjacent side to } \theta}{\text{opposite side to } \theta}$$

$$14) \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side to } \theta}$$

$$15) \cosec\theta = \frac{\text{Hypotenuse}}{\text{Opposite side to } \theta}$$

$$16) \sin\theta \times \cosec\theta = 1$$

$$17) \cos\theta \times \sec\theta = 1$$

$$18) \tan\theta \times \cot\theta = 1$$

$$19) \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$20) \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$21) \sin^2\theta + \cos^2\theta = 1$$

$$22) \sec^2\theta - \tan^2\theta = 1$$

$$23) \cosec^2\theta - \cot^2\theta = 1$$

$$24) \sin\theta = \sqrt{1 - \cos^2\theta}$$

$$25) \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$26) \tan\theta = \sqrt{\sec^2\theta - 1}$$

$$27) \sec\theta = \sqrt{1 + \tan^2\theta}$$

$$28) \cot\theta = \sqrt{\cosec^2\theta - 1}$$

$$29) \cosec\theta = \sqrt{1 + \cot^2\theta}$$

$$30) -1 \leq \sin\theta \leq 1$$

$$31) -1 \leq \cos\theta \leq 1$$

$$32) -\infty < \tan\theta < \infty$$

$$33) \text{ If } \sec\theta + \tan\theta = a \text{ then } \sec\theta - \tan\theta = \frac{1}{a}, (a \neq 0)$$

$$34) \text{ If } \cosec\theta + \cot\theta = a \text{ then } \cosec\theta - \cot\theta = \frac{1}{a}, (a \neq 0)$$

$$35) \text{ In British system } 1^0 = 60^1$$

$$1^1 = 60^{11}$$

$$36) \text{ In French system } 1^g = 100^1$$

$$1^1 = 100^{11}$$

37)

$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$135^\circ$	$180^\circ$	$360^\circ$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{1}{\sqrt{2}}$	0	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{2}}$	-1	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	-1	0	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	$-\sqrt{2}$	-1	1
$\cosec\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\sqrt{2}$	Not defined	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	-1	Not defined	Not defined

$$38) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$39) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$40) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$41) \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$42) \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$43) \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$44) \cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$45) \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$46) \sin(A+B) \sin(A-B) = \sin^2 A - \sin^2 B$$

$$= \cos^2 B - \cos^2 A$$

$$47) \cos(A+B) \cos(A-B) = \cos^2 A - \sin^2 B$$

$$= \cos^2 B - \sin^2 A$$

$$48) \tan\left(\frac{\pi}{4} + A\right) = \frac{1+\tan A}{1-\tan A}$$

$$49) \tan\left(\frac{\pi}{4} - A\right) = \frac{1-\tan A}{1+\tan A}$$

$$50) \cot\left(\frac{\pi}{4} + A\right) = \frac{\cot A - 1}{\cot A + 1}$$

$$51) \cot\left(\frac{\pi}{4} - A\right) = \frac{\cot A + 1}{\cot A - 1}$$

$$52) \sin 2A = 2\sin A \cos A$$

$$= \frac{2\tan \theta}{1+\tan^2 A}$$

$$53) \cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 2\sin^2 A$$

$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$54) \tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$55) \sin \theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$56) \cos \theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$57) \tan \theta = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$$

58)

$\theta$	$15^0$	$75^0$
Sin	$\frac{\sqrt{3} - 1}{2\sqrt{2}}$	$\frac{\sqrt{3} + 1}{2\sqrt{2}}$
Cos	$\frac{\sqrt{3} + 1}{2\sqrt{2}}$	$\frac{\sqrt{3} - 1}{2\sqrt{2}}$
Tan	$2 - \sqrt{3}$	$2 + \sqrt{3}$

59)

$\theta$	$18^0$	$36^0$	$54^0$	$72^0$
Sin	$\frac{\sqrt{5} - 1}{4}$	$\frac{\sqrt{10 - 255}}{4}$	$\frac{\sqrt{5} + 1}{4}$	$\frac{\sqrt{10 + 255}}{4}$
Cos	$\frac{\sqrt{10 + 255}}{4}$	$\frac{\sqrt{5} + 1}{4}$	$\frac{\sqrt{10 - 255}}{4}$	$\frac{\sqrt{5} - 1}{4}$
Tan	$\sqrt{4\sqrt{5} - 8}$	$\sqrt{5 - 2\sqrt{5}}$	$\sqrt{4\sqrt{5} + 8}$	$\sqrt{5 + 2\sqrt{5}}$

60)

$\theta$	$9^0$	$22\frac{1}{2}^0$	$67\frac{1}{2}^0$
Sin	$\frac{\sqrt{3 + \sqrt{5}} - \sqrt{5 - \sqrt{5}}}{4}$	$\frac{\sqrt{2 - \sqrt{2}}}{2}$	$\frac{\sqrt{2 + \sqrt{2}}}{2}$
Cos	$\frac{\sqrt{3 + \sqrt{5}} + \sqrt{5 - \sqrt{5}}}{4}$	$\frac{\sqrt{2 + \sqrt{2}}}{2}$	$\frac{\sqrt{2 - \sqrt{2}}}{2}$
Tan	$(\sqrt{5} + 1) - \sqrt{5 + 2\sqrt{5}}$	$\sqrt{2} - 1$	$\sqrt{2} + 1$

$$61) \sin A = 2 \sin A/2 \cos A/2$$

$$= \frac{2 \tan A/2}{1 + \tan^2 A/2}$$

$$62) \cos A = \cos^2 A/2 - \sin^2 A/2$$

$$= 2 \cos^2 A/2 - 1$$

$$= 1 - 2 \sin^2 A / 2$$

$$= \frac{1 - \tan^2 A / 2}{1 + \tan^2 A / 2}$$

$$63) \tan A = \frac{2 \tan A / 2}{1 - \tan^2 A / 2}$$

$$64) \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$65) \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$66) \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$67) \cot 3A = \frac{3 \cot A - \cot^3 A}{1 - 3 \cot^2 A}$$

$$68) \tan\left(\frac{\pi}{4} + \theta\right) = \cot\left(\frac{\pi}{4} - \theta\right) = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

$$69) \tan\left(\frac{\pi}{4} - \theta\right) = \cot\left(\frac{\pi}{4} + \theta\right) = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$70) \sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$$

$$71) \cos \theta \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$$

$$72) \tan \theta \tan(60^\circ - \theta) \tan(60^\circ + \theta) \tan 3\theta$$

$$73) \cot \theta \cot(60^\circ - \theta) \cot(60^\circ + \theta) = \cot 3\theta$$

$$74) \cot(A + B) \cot(A - B) = \frac{\cot^2 A \cot^2 B - 1}{\cot^2 B - \cot^2 A}$$

$$75) \tan(A + B) \tan(A - B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

$$76) \sin(A + B) + \sin(A - B) = 2 \sin A \cos B$$

$$77) \sin(A + B) - \sin(A - B) = 2 \sin A \sin B$$

$$78) \cos(A + B) + \cos(A - B) = 2 \cos A \cos B$$

$$79) \cos(A + B) - \cos(A - B) = -2 \sin A \sin B$$

$$80) \tan A + \sin A = m, \tan A - \sin A = n \text{ then } m^2 - n^2 = 4\sqrt{mn}$$

$$81) \sin(-\theta) = -\sin \theta$$

$$82) \cos(-\theta) = -\cos\theta$$

$$83) \tan(-\theta) = -\tan\theta$$

$$84) \cot(-\theta) = -\cot\theta$$

$$85) \sec(-\theta) = \sec\theta$$

$$86) \csc(-\theta) = -\csc\theta$$

87)  $\cos x + \cos y = a, \sin x + \sin y = b$  then

$$a) \sin(x+y) = \frac{2ab}{a^2+b^2}$$

$$b) \cos(x+y) = \frac{a^2-b^2}{a^2+b^2}$$

$$c) \tan(x+y) = \frac{2ab}{a^2-b^2}$$

$$d) \tan\frac{x+y}{2} = \frac{b}{a}$$

88)  $\cos x - \cos y = a, \sin x - \sin y = b$  then

$$a) \sin(x+y) = \frac{-2ab}{a^2+b^2}$$

$$b) \cos(x+y) = \frac{b^2-a^2}{b^2+a^2}$$

$$c) \tan(x+y) = \frac{-2ab}{a^2-b^2}$$

$$d) \tan\frac{(x+y)}{2} = \frac{-a}{b}$$

$$89) \cos\theta + \cos(120^\circ - \theta) + \cos(120^\circ + \theta) = 0$$

$$90) \sin\theta + \sin(120^\circ + \theta) - \sin(120^\circ - \theta) = 0$$

91) If  $A+B=45^\circ$  or  $225^\circ$  then

$$(1 + \tan A)(1 + \tan B) = 2$$

$$(1 - \cot A)(1 - \cot B) = 2$$

92) If  $A+B=135^0$  or  $315^0$  then

$$(1 - \tan A)(1 - \tan b) = 2$$

$$(1 + \cot A)(1 + \cot B) = 2$$

93) If  $A+B+C = 180^0$  then

$$a) \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$b) \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

94) If  $A+B+C=90^0$  then

$$a) \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$$

$$b) \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

95)  $\cot A + \tan A = 2 \operatorname{cosec} 2A$

96)  $\cot A - \tan A = 2 \cot 2A$

97)  $a \cos\theta + b \sin\theta = c$  then

$$a \sin\theta - b \cos\theta = \pm \sqrt{a^2 + b^2 - c^2}$$

98)  $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta \cos^2\theta$

99) If  $\sin\theta + \cos\theta = a$  then  $\sin\theta - \cos\theta = \sqrt{2 - a^2}$

	$90-\theta$	$90+\theta$	$180-\theta$	$180+\theta$	$270-\theta$	$270+\theta$	$360-\theta$	$360+\theta$
Sin	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
Cos	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
Tan	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$

Cot	Tanθ	-Tanθ	-Cotθ	Cotθ	Tanθ	-Tanθ	-Cotθ	Cotθ
Sec	Cosecθ	-Cosecθ	-Secθ	-Secθ	-Cosecθ	Cosecθ	Secθ	Secθ
Cosec	Secθ	Secθ	Cosecθ	-Cosecθ	-Secθ	-Secθ	-Cosecθ	Cosecθ

$$100) \sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$$

$$101) \text{ If } \frac{\pi}{4} < \frac{A}{2} < \frac{3\pi}{4} \text{ then}$$

$$2 \sin A/2 = \sqrt{1 + \sin A} + \sqrt{1 - \sin A}$$

$$102) \text{ If } \frac{3\pi}{4} < \frac{A}{2} < \frac{5\pi}{4}$$

$$2 \sin A/2 = -\sqrt{1 + \sin A} + \sqrt{1 - \sin A}$$

$$2 \cos A/2 = -\sqrt{1 + \sin A} - \sqrt{1 - \sin A}$$

$$103) \text{ If } \frac{5\pi}{4} < \frac{A}{2} < \frac{7\pi}{4}$$

$$2 \sin A/2 = -\sqrt{1 + \sin A} + \sqrt{1 - \sin A}$$

$$2 \cos A/2 = -\sqrt{1 + \sin A} + \sqrt{1 - \sin A}$$

$$104) \text{ If } \frac{-\pi}{4} < \frac{A}{2} < \frac{\pi}{4} \text{ then}$$

$$2 \sin A/2 = -\sqrt{1 + \sin A} + \sqrt{1 - \sin A}$$

$$2 \cos A/2 = -\sqrt{1 + \sin A} + \sqrt{1 - \sin A}$$

$$105) \sin 4A = 4 \sin A \cos A - 8 \sin^3 A \cos A$$

$$106) \cos 4A = 8 \cos^4 A - 8 \cos^2 A + 1$$

$$107) \tan 4A = \frac{4 \tan A - 4 \tan^3 A}{1 - 6 \tan^2 A + \tan^4 A}$$

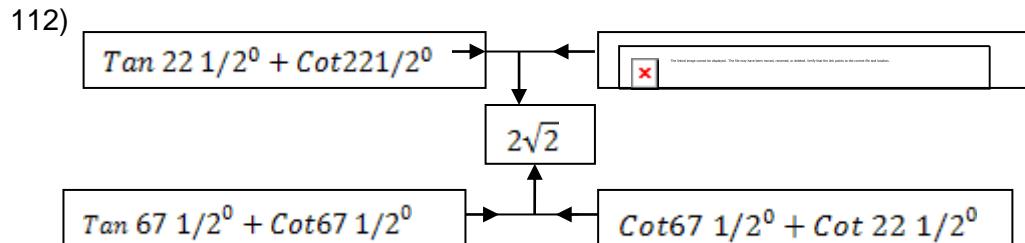
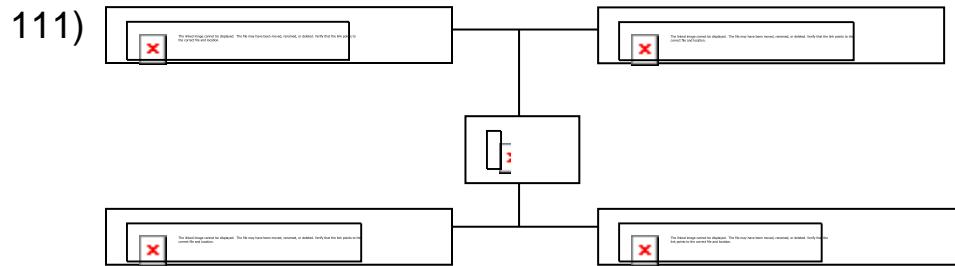
$$108) \sin^3 A = \frac{1}{4} [3 \sin A - \sin 3A]$$

$$109) \cos^3 A = \frac{1}{4}[3\cos A + \cos 3A]$$

$$110) \text{ a)} \sin 71\frac{1}{2}^\circ = \frac{\sqrt{2\sqrt{2}-\sqrt{3}-1}}{4\sqrt{2}}$$

$$\text{b)} \cos 71\frac{1}{2}^\circ = \frac{\sqrt{2\sqrt{2}+\sqrt{3}+1}}{4\sqrt{2}}$$

$$\text{c)} \tan 71\frac{1}{2}^\circ = \sqrt{2} - \sqrt{3} - \sqrt{4} + \sqrt{6}$$



$$113) \cos^3 \theta + \cos^3(120^\circ - \theta) + \cos^3(120 + \theta) = \frac{3}{4} \cos 3\theta$$

$$114) \cos^3 \theta + \cos^3(240^\circ - \theta) + \cos^3(240 + \theta) = \frac{3}{4} \cos 3\theta$$

$$115) \cos^3 \theta + \cos^3(480^\circ - \theta) + \cos^3(480 + \theta) = \frac{3}{4} \cos 3\theta$$

$$116) \cos^3 \theta + \cos^3(600^\circ - \theta) + \cos^3(600^\circ + \theta) = \frac{3}{4} \cos 3\theta$$

$$117) \cos(45^\circ - A)\cos(45^\circ - B) - \sin(45^\circ - A)\sin(45^\circ - B) = \sin(A + B)$$

$$118) \sin(45^\circ + A)\cos(45^\circ - B) + \cos(45^\circ + A)\sin(45^\circ - B) = \cos(A - B)$$

$$119) \tan(A + B) = \frac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B}$$

$$120) \sin^2 \theta + \sin^2(60^\circ - \theta) + \sin^2(60^\circ + \theta) = \frac{3}{2}$$

121) Triplets for a right angle triangle

- |           |            |
|-----------|------------|
| (3,4,5)   | (14,48,50) |
| (6,8,10)  | (10,24,26) |
| (7,24,25) | (12,16,20) |
| (5,12,13) | (21,72,75) |
| (9,12,15) | (15,36,39) |

$$122) \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$123) \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$124) \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$125) \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

126) Area of triangle

$$\begin{aligned} &= \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C \\ &= 2R^2 \sin A \sin B \sin C \\ &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \frac{abc}{4R} \end{aligned}$$

$$127) \text{Area of triangle} = \sqrt{2(a^2b^2 + b^2c^2 + c^2a^2) - (a^4 + b^4 + c^4)}$$

128) In  $\Delta ABC$ ,  $\sin(A+B) = \sin C$

$$\sin(B+C) = \sin A$$

$$\sin(C+A) = \sin B$$

129) In  $\Delta ABC$ ,  $\cos(A+B) = -\cos C$ ,  $\tan(A+B) = -\tan C$

$$\cos(B+C) = -\cos A$$

$$\tan(B+C) = -\tan A$$

$$\cos(C+A) = -\cos B$$

$$\tan(C+A) = -\tan B$$

$$130) \sin \theta = \sin \theta$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$$

$$\sec \theta = \frac{1}{\sqrt{1 - \sin^2 \theta}}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta}$$

$$131) \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\cos \theta = \cos \theta$$

$$\tan \theta = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sqrt{1 - \cos^2 \theta}}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sqrt{1 - \cos^2 \theta}}$$

$$132) \sin \theta = \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$$

$$\cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}}$$

$$\tan \theta = \tan \theta$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sec \theta = \sqrt{1 + \tan^2 \theta}$$

$$\csc \theta = \frac{\sqrt{1 + \tan^2 \theta}}{\tan \theta}$$

$$133) \sin \theta = \frac{1}{\sqrt{1 + \cot^2 \theta}}$$

$$\cos \theta = \frac{\cot \theta}{\sqrt{1 + \cot^2 \theta}}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \cot \theta$$

$$\sec \theta = \frac{\sqrt{1 + \cot^2 \theta}}{\cot \theta}$$

$$\cosec \theta = \sqrt{1 + \cot^2 \theta}$$

$$134) \sin \theta = \frac{\sqrt{\sec^2 \theta - 1}}{\sec \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$\cot \theta = \frac{1}{\sqrt{\sec^2 \theta - 1}}$$

$$\cosec \theta = \frac{\sec \theta}{\sqrt{\sec^2 \theta - 1}}$$

$$135) \sin \theta = \frac{1}{\cosec \theta}$$

$$\cos \theta = \frac{\sqrt{\cosec^2 \theta - 1}}{\cosec \theta}$$

$$\tan \theta = \frac{1}{\sqrt{\cosec^2 \theta - 1}}$$

$$\cot \theta = \sqrt{\cosec^2 \theta - 1}$$

$$\sec \theta = \frac{\cosec \theta}{\sqrt{\cosec^2 \theta - 1}}$$

$$\cosec \theta = \cosec \theta$$

136) Maximum value of  $a \cos x + b \sin x + c$  is  $c + \sqrt{a^2 + b^2}$

Maximum value of  $a \cos x + b \sin x + c$  is  $c - \sqrt{a^2 + b^2}$

## 10) STATISTICS

1) Arithmetic mean of individual series  $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{Ex}{n}$

2) Arithmetic mean of discrete series

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{f_1 + f_2 + f_3 + \dots + f_n} = \frac{\sum f_i x_i}{\sum f_i}$$

3) Weighted arithmetic mean =  $\frac{x_1 w_1 + x_2 w_2 + \dots + x_n w_n}{w_1 + w_2 + \dots + w_n} = \frac{Ex;w}{Ew}$

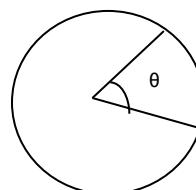
4) Mean of first 'n' natural numbers =  $\frac{n+1}{2}$

5) Mean of squares of first 'n' natural numbers =  $\frac{(n+1)(2n+1)}{6}$

6) A.M =  $A + \frac{\sum f_i v_i}{N} \times C$  (deviation method)

7) The mean of first 'n' odd natural numbers is  $n$

8) The mean of first 'n' even natural numbers is  $n+1$

- 9) The A.M of a set of observation is  $\bar{x}$ . if each observation is divided by 'a' and then is increased by 'b' the mean of new one is  $\frac{\bar{x}}{a} + b$
- 10) The A.M of 'n' items of a data is 'm'. then the sum of the values of the items is  $mn$
- 11) The Range of first 'n' natural numbers is  $n-1$
- 12) If  $x_1, x_2, x_3, \dots, x_n$  are in ascending order then median is  $\left(\frac{n+1}{2}\right)^{th}$  term if n is odd
- 13) Median =  $l + \frac{\left(\frac{N}{2}-F\right)}{f} \times C$
- 14) Mode =  $l_1 + \frac{\Delta_1}{\Delta_1+\Delta_2} \times C, \Delta_1 = f - f_1, \Delta_2 = f - f_2$
- 15) Mode =  $(3 \times \text{Median}) - (2 \times \text{mean})$
- 16) If Mean = x, median = x then mode = x
- 17) For symmetric distribution Mean = Median = Mode
- 18) Sector angle =  $\frac{\text{Component value}}{\text{Total value}} \times 360^\circ$   
 (In the construction of pie diagram)
- 
- 19) Mode of first 'n' natural numbers does not exists
- 20) The difference in extreme values of a data is called range
- 21) Mean  $(ax_i + b) = a$  mean  $(x_i) + b$
- 22)  $Q D = \frac{Q_3 - Q_1}{2}$
- 23) Variance =  $\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}$  ( $\bar{x} = \text{Mean}$ )
- 24) SD =  $\sqrt{\text{Variance}}$

## (11) MATRICES

1. In matrices  $(A^T)^T = A$

2.  $(A + B)^T = A^T + B^T$
3.  $(A - B)^T = A^T - B^T$
4.  $(AB)^T = B^T \cdot A^T$
5.  $(ABC)^T = C^T \cdot B^T \cdot A^T$
6.  $(A + B + C)^T = A^T + B^T + C^T$
7.  $AB^T = (BA^T)^T$
8.  $(AB^T)^T = BA^T$
9.  $(A^2)^T = (A^T)^2$
10.  $A + B^T = (A^T + B)^T$
11.  $C = (A)_{m \times n} \times (B)_{n \times p}$  then order of C is  $m \times p$
12.  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  then  $I^n = I$
13.  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $|A| = ad - bc$
14. If  $A = A^T$  then A is called symmetric matrix
15. If  $A = A^T$  then A is called skew symmetric matrix

### Symmetric:-

- If A, B are symmetric matrices then
16.  $A+B$  symmetric
  17.  $A-B$  symmetric
  18.  $AB + BA$  symmetric
  19.  $A^2, B^2$  symmetric
  20.  $A^2 + B^2, A^2 - B^2$  symmetric
  21.  $AB, BA$  not symmetric

### Skew Symmetric:-

- If A, B are skew symmetric then
22.  $A + B, A - B$  are skew symmetric
  23.  $A^2, B^2$  are symmetric
  24.  $A^2 + B^2, A^2 - B^2$  are symmetric
  25.  $A^2, A^4, A^6, \dots$  are symmetric
  26.  $A^3, A^5, A^7, \dots$  are skew symmetric
  27. If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
  28.  $AA^{-1} = A^{-1} \cdot A = I$
  29.  $(AT)^{-1} = (A^{-1})^T$
  30.  $|A| = |A^T|$
  31.  $I^{-1} = I$

32.  $(AB)^{-1} = B^{-1}, A^{-1}$

33. If  $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$  then  $A+B, A-B, A^2, A^T$  are also diagonal matrices

34. If  $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$  then  $A^n = \begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$

35.  $A = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$  then  $A^n = a^n \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$

36.  $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$  then  $\text{trace}(A) = a + b$

37.  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$  then  $\text{trace}(A) = a + b + c$

38. In crammers method

$$x = \frac{|B_1|}{|A|}, y = \frac{|B_2|}{|A|}$$

39. In matrix inversion method

$$x = (y^x) = A^{-1}B$$

40. If  $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0, a_3x + b_3y + c_3 = 0$ , then area of triangle =

$$\frac{1}{2} \left| \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \right|^2$$

$$\frac{\left| a_1 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \times a_2 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \times a_3 \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} \right|}{\left| a_1 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \times a_2 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \times a_3 \begin{vmatrix} b_3 & c_3 \\ b_1 & c_1 \end{vmatrix} \right|}$$

41. If  $(A)_{n \times m}, (B)_{n \times m}$  then both  $AB, BA$  exists

42. Matrix addition is closure, commutative, associative

43. In matrices i)  $AB \neq BA$

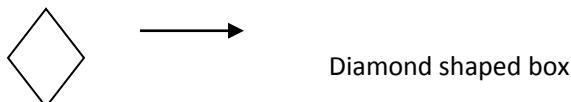
ii)  $(AB)C = A(BC)$

44.  $(KA)^T = KA^T$

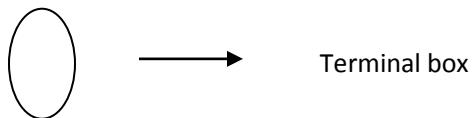
## 12) COMPUTING



Rhombus shaped box (or)



Diamond shaped box



→ Input – out put box

□ → Data box

□ → Decision box

A computer has three major components

1) Input and output devices

2) Central processing unit

3) Memory or storage

**PROGRAMME**:- The method of solving a problem

**FLOWCHARTS**:- The diagrammatic or pictorial representation describing a method of solving a problem

**ALGORITHM**:- A technical term for a step by step procedure specifying a sequence of steps resulting in the computation of a task or a job

**COMPUTER LANGUAGES**:- Fortran, Basic, Cobol, Pascal,

**BINARY SYSTEM**:- In this system we use only 0 and 1 digits

**Binary system**

0

1

10

11

100

101

**Decimal system**

0

1

2

3

4

5

110	6
111	7
1000	8
1001	9

1 Nibble = 4 bits

1 MB = 1024 KB

1 GB = 1024 MB

1 Byte = 8 bits

1 Kilobyte=1024 bytes = bytes

1 GB = 1024 MB

1 TB = 1024 GB

### 13) LOGATITHMS

$$1) \text{ If } a^x = N \Rightarrow x = \log_a^N$$

$$2) \log_x ab = \log_x^a + \log_x^b$$

$$3) \log_x \left(\frac{a}{b}\right) = \log_x^a - \log_x^b$$

$$4) \log a^m = m \log a$$

$$5) \log_b^a = \frac{\log a}{\log b}$$

$$6) \log_b^a = \frac{1}{\log_a^b}$$

$$7) \log_b^a = \log_x^a \times \log_b^x$$

$$8) \log_a^a = 1$$

$$9) \log_a^0 = -\infty (a > 1)$$

$$10) \log_a^0 = +\infty (a < 1)$$

$$11) \log_a^\infty = \infty (a > 1)$$

$$12) a^{\log_a^m} = m$$

$$13) \log_b^a \times \log_c^b \times \log_a^c = 1$$

$$14) \log_{ab} x = \frac{\log_b^x}{1 + \log_a^b}$$

$$15) a^{\log_a^m + \log_a^n} = mn, (a \neq 1)$$

$$16) a^{\log_a^m - \log_a^n} = m/n, (a \neq 1)$$

$$17) a^{\log_b^m + \log_a^n} = m$$

$$18) (a^2)^{\log_a^x} = x^2$$

$$19) a^{\log_{\sqrt{a}}^x} = x^2$$

$$20) a^{-\log_a^x} = \frac{1}{x}$$

$$21) (a^2)^{-\log_a^x} = \frac{1}{x^2}$$

$$22) (\sqrt{a})^{\log_a^x} = \sqrt{x}$$

$$23) \frac{1}{\log_a^{ab}} + \frac{1}{\log_a^{ab}} = 1$$

$$24) \frac{1}{\log_a^{abc}} + \frac{1}{\log_b^{abc}} + \frac{1}{\log_c^{abc}} = 1$$

$$25) \frac{1}{\log_{\sqrt{a}}^{abc}} + \frac{1}{\log_{\sqrt{b}}^{abc}} + \frac{1}{\log_{\sqrt{c}}^{abc}} = \frac{1}{2}$$

$$26) \frac{1}{\log_{\frac{1}{a}}^{abc}} + \frac{1}{\log_{\frac{1}{b}}^{abc}} + \frac{1}{\log_{\frac{1}{c}}^{abc}} = -1$$

$$27) \log(\log x^p) - \log(\log x^q) = \log(\frac{p}{q})$$

$$28) \log(\log x^{1/p}) - \log(\log x^{1/q}) = \log(\frac{q}{p})$$

$$29) \frac{\log_a^x}{\log_a^y} - \frac{\log_b^x}{\log_b^y} = 0$$

30) If a number has 'n' digits the characteristic of its logarithm is n-1

31) If the characteristic of the logarithm of a number is 'n', the number will have 'n+1' digits

32) If in a decimal fraction there are 'n' zeros after decimal point and before the significant number, the characteristic of that. Logarithm is  $\overline{n+1}$

33) The characteristic is always an integer. It may be positive or negative or zero

$$34) \log_a^1 = 0$$

$$35) \log_y^x = \frac{\log_y^a}{\log_x^a}$$

$$36) \log_y^x = \frac{\log_a^x}{\log_a^y}$$

$$37) a^x = b^y \Rightarrow \log_b a = \frac{y}{x}$$

$$38) \log_a(x - k) < \log_{a^2}(r - k) \Rightarrow x \in (k, k + 1)$$

$$39) \text{ If } x = \frac{a^{-n} - a^{-n}}{a^n + a^{-n}} \text{ then } n = \frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$$

$$40) \log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$41) \log(1 - x) = x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$42) \text{ If } a > 0, x_1 < x_2 \text{ then } \log_a^{x_1} < \log_a^{x_2}$$

$$43) \text{ If } 0 < a < 1, x_1 < x_2 \text{ then } \log_a^{x_1} > \log_a^{x_2}$$

#### 14) MENSURATION

$$1) \text{ Area of a square} = S^2 = s \times s$$

$$2) \text{ Area of a rectangle} = l \times b$$

$$3) \text{ Area of a triangle} = \frac{1}{2}bh$$

$$4) \text{ Area of a parallelogram} = bh$$

$$5) \text{ Area of a rhombus} = \frac{1}{2}d_1d_2$$

$$6) \text{ Area of a trapezium} = \frac{1}{2}h(a + b)$$

$$7) \text{ Area of a quadrilateral} = \frac{1}{2}d(h_1 + h_2)$$

$$8) \text{ Area of a circle} = \pi r^2$$

$$9) \text{ Area of semi circle} = \frac{\pi r^2}{2}$$

$$\frac{\pi d^2}{8}$$

$$10) \text{ Area of square} = \frac{1}{2}(\text{diagonal})^2$$

$$11) \text{ Area of ring} = \pi(R + r)(R - r)$$

$$12) \text{ Total surface area of a cube} = 6s^2$$

$$13) \text{ Lateral surface area of a cube} = 4s^2$$

$$14) \text{ Total surface area of a cuboid} = 2(lb + bh + lh)$$

$$15) \text{ Lateral surface area of a cuboid} = 2h(l + b)$$

16) Area of 4 walls =  $2h(l + b)$

$$= ph$$

$$= 4lh$$

17) Perimeter of a square =  $4 \times \infty = a\sqrt{A}$

18) Perimeter of a rectangle =  $2(l + b)$

19) Perimeter of a parallelogram =  $2(l + b)$

20) Circumference of a circle =  $2\pi r$

$$= \pi d$$

21) Circumference of a semicircle =  $\frac{36r}{7}$

22) Circumference of a sector =  $l + 2r$

23) Area of a sector =  $\frac{x}{360^\circ} \times \pi r^2$

24) Diagonal of a rectangle =  $\sqrt{l^2 + b^2}$

25) Diagonal of a square =  $\sqrt{258}$

26) Diagonal of a cuboid =  $\sqrt{l^2 + b^2 + h^2}$

27) Diagonal of a cube =  $\sqrt{3} s$

28) Sum of the edges of a cuboid =  $4(l + b + h)$

29) No. of faces of a prism = no. of lateral surfaces + base + top

30) Volume of the prism = Area of the base X height

31) Lateral surface area of prism = (perimeter of the base) X (height)

32) Volume of water flowing through sluice

$$= \text{Area of cross section of sluice} \times \text{speed of water flowing per hour} .$$

33) Box with lid

- i) Inner length = outer length - 2 X thickness of wood
- ii) Inner breadth = outer breadth - 2 X thickness of wood
- iii) Inner height = outer height - 2 X thickness of wood

34) Box without lid

- iv) Inner length = l - 2t
- v) Inner breadth = b - 2t
- vi) Inner height = h - t

35) Volume of masonry = Area of cross section X length

36) Area of cross section of masonry = Area of rectangle + Area of semi circle

37) Rise in level =  $\frac{\text{volume}}{\text{Area}}$

- 38) Volume of right circular cylinder =  $\pi r^2 h$   
 39) Lateral surface area of cylinder =  $2\pi r h$   
 40) Each base surface area of cylinder =  $\pi r^2$   
 41) Total surface area of cylinder =  $2\pi r(r + h)$   
 42) Volume of hallow cylinder =  $\pi(R + r)(R - r)h$   
 43) Area of base of a cone =  $\pi r^2$   
 44) Curved surface area of a cone =  $\pi r l$   
 45) Total surface area of a cone =  $\pi r(l + r)$   
 46) Volume of a cone =  $\frac{1}{3} \pi r^2 h$   
 47) 'L' is slant height, r is base radius, h be vertical height of a cone then =  $l^2 = r^2 + h^2$   
 48) Surface area of a sphere =  $4\pi r^2$   
 49) Volume of a sphere =  $\frac{4}{3} \pi r^3$   
 50) Curved surface area of hemi sphere =  $2\pi r^2$   
 51) Total surface area of hemi sphere =  $3\pi r^2$   
 52) Volume of hemi sphere =  $\frac{2}{3} \pi r^3$   
 53) Volume of hollow sphere =  $\frac{4}{3} \pi (R^3 - r^3)$   
 54) Total surface area of a hemi spherical vessel =  $\pi(3R^2 + r^2)$   
 55) The area of the ring around the top of the hemi spherical vessel =  $\pi(R^2 - r^2)$   
 56) The surface areas of two spheres are in the ratio of the squares of their radii.  
 57) 1 Km = 1000 , mts  
 58) 1 Km = 100000 cm  
 59) 1 Km = 1000000 mm  
 60) 1 Decimeter = 10 mtss  
 61) 1 Decimeter = 1/10 mts  
 62) 1 Cm = 10 mm  
 63) 1 Cm = 1/100 mts  
 64) 1 m = 100 cm  
 65) 1 m = 1000 mm  
 66) 1 square meter = 10,000 sq. centimeters  
 67) 1 Area = 1 sq. decameter  
       = 100 sq.m  
 68) 1 Hectare = 10,000 square meters

- 69) 1 Sq.kilometer = 100 hectares  
 70) 1 Quintal = 100 Kg  
 71) 1 Tonne = 1000 Kg  
 72) 1 Ream = 40 dozens = 480 sheets  
 73) 1 Gross = 144  
 74) Representative ratio =  $\frac{\text{Distance in sketch}}{\text{corresponding distance on ground}}$ :  
 75) Representative fraction =  
 76) Volume = Area X height  
 77) 1 Quire = 24 sheets  
 78) 1 Ream = 20 quires

## **15) BUSINESS MATHEMATICS**

- 1) If  $a : b = c : d$  then  $ad = bc$   
 2) If  $b$  is mean proportional between  $a, c$  then  
 3) If  $x \propto y$  then  $x = ky$   
 4) If  $x \propto \frac{1}{y}$  then  $xy = k$   
 5) If there is an increase of  $x\%$  in a quantity, then its value increases by  $(100 + x)\%$  over its original value. Multiplying fraction to get increased value is  $\frac{100+x}{100}$   
 6) If there is a decrease of  $x$  in a quantity then its value decreases by  $(100 - x)\%$  over its original value multiplying fraction to get the decreased value is  $\frac{100-x}{100}$   
 7) Profit = selling price – cost price  
 8) Loss = Cost price – selling price  
 9) Percentage profit =  $\frac{p}{cp} \times 100$   
 10) Percentage loss =  $\frac{l}{cp} \times 100$   
 11)  $S.P = C.P \times \frac{100+g}{100}$   
 12)  $S.P = C.P \times \frac{100-l}{100}$   
 13)  $C.P = S.P \times \frac{100}{100+g}$   
 14)  $C.P = S.P \times \frac{100}{100-l}$   
 15) Simple interest ( $I$ ) =  $\frac{PTR}{100}$

$$16) \text{ Principal (P)} = \frac{100 \times I}{TR}$$

$$17) \text{ Time (T)} = \frac{100 \times I}{PR}$$

$$18) \text{ Rate (R)} = \frac{100 \times I}{PT}$$

$$19) \text{ Amount (A)} = P \left(1 + \frac{TR}{100}\right)$$

$$= P + I$$

20) Compound interest (C.I) = A - P

$$= P \left(1 + \frac{R}{100}\right)^N - P$$

21) If an article is sold at a gain of 10% then S.P = 110% of C.P

22) If an article is sold at a loss of 10% then S.P = 90% C.P

23) If A's salary is r more than B's then B's salary less than A is  $\left[\frac{r}{100+r} \times 100\right]\%$

24) If A's salary r less than B's salary. The B's salary above A's is  $\left[\frac{r}{100-r} \times 100\right]\%$

25) S.P = Market price – discount

$$26) \text{ S.P} = \text{M.P} \frac{100-d}{100}$$

$$27) D\% = \frac{d}{M.P} \times 100$$

$$28) \text{ Average} = \frac{\text{Sum of quantities}}{\text{No.of quantities}}$$

$$29) \text{ Average speed} = \frac{2xy}{x+y}$$

30) If A can do a piece of work in 'n' days, then the work done by A in one day =  $(1/n)$

31) If the no. of men be changed in the ratio m:n, the time taken to finish the work will be changed in the ratio n:m

32) If a:b = c:d then B:A = D:C

33) If a:b = c:d then a + b : b = c + d : d

34) If a:b = c:d then a – b : b = c – d : d

35) If a:b = c:d then a + b : a - b = c + d : c - d

## 16) NUMBERS

1) I.C.M X G.C.D = product of two numbers

2) HCF (or) G.C.D of two co primes = 1

3) H.C.F of fractions = 
$$\frac{H.C.F \text{ of numerators}}{L.C.M \text{ of Denominators}}$$

*L.C.M of Denominators*

- 4) L.C.M of fractions =  $\frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$
- 5) Generally two digit number in the form of  $10x + 10y + z$
- 6) Generally three digit number in the form of  $100x + 10y + z$  Type equation here.
- 7)  $n^2 + n + 41$  is a prime if  $n < 40$
- 8)  $n^2 + n + 17$  is a prime if  $n < 16$
- 9)  $2n^2 + 29$  is a prime if  $n < 29$
- 10)  $n^2 - 79n + 1601$  is a prime if  $n < 80$
- 11)  $2^{2n} + 1$  is a prime if  $n < 5$
- 12) If 'n' is even  $n(n^2 + 20)$  divisible by 48
- 13) If 'n' is a prime, greater than 3,  $n^2 - 1$  is divisible by 24
- 14)  $[x(x^{n-1} - n^{an-1}) + a^n(n-1)]$  is divisible by  $(x-a)^2$ , if  $n > 1$