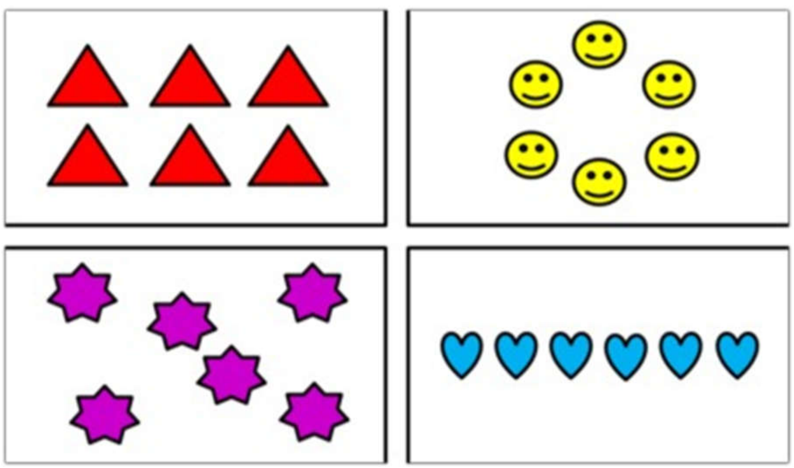
SETS

1.Sets

- Definition of sets
- Roster form & Set builder form
- Void or empty set
- Finite set & infinite set
- Equal set
- Subsets, Proper sets & Supersets
- Singleton set , Power set & Universal sets
- Open & Closed intervals
- Venn diagrams
- Operations on sets: Union, Intersection, Difference
- Complement of set

Definition: Set

- Set is a well-defined collection of objects.
- Ex. Members of a family, Students of ABC school, Players of Indian cricket team, Rivers of India.



Sets in Mathematical world

N : the set of all natural numbers

Z : the set of all integers

Q : the set of all rational numbers

R : the set of real numbers

Z^+: the set of positive integers

Q^+: the set of positive rational numbers

R^+: the set of positive real numbers.

Conventions in set:

- Sets are usually denoted by capital letters A,B,C,X,Z etc.
- The elements of set are represented by small letters a,b,c,x,z etc.
- If 'a' is an element of a set A, we say "a belongs to A" , the Greek symbol '∈'(epsilon) is used to denote the phrase 'belongs to'. Thus we write $a \in A$.
- If 'b' is not an element of a set A, we write $b \notin A$, and read 'b does not belong to A'.
- Objects, members or elements of a set are synonymous terms.

Methods for representing a set -

- Roster form
- Set-builder form

- Roster form: (All elements listed)

In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.

E.g. The set of all vowels in the English alphabet is {*a, e, i , o, u*}

Note :

1.In roster form, the order in which the elements are listed is immaterial.
Thus, the above set can also be represented as {u , o , *a, e, i* }.

2. While writing the set in roster form an element is not generally repeated, i.e., all the elements are taken as distinct. For example, the set of letters forming the word 'SCHOOL' is { S, C, H, O, L} or {H, O, L, C, S}.

- Set-builder form: (Rule or pattern)

In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

E.g. In the set {a, e, i, o, u}, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property.

Denoting this set by V, we write V= {x : x is a vowel in English alphabet}

Note :

1.Any other symbol like the letters y, z, etc. could be used instead of x.

2.The symbol should be followed by colon :

3.After the sign of colon, we write the characteristic property possessed by the elements of the set and then enclose the whole description within braces.

*Imp. – If a set doesn't follow a pattern it can't be written in set-builder form.

Empty or void or null set -

- A set which does not contain any element is called the *empty set* or the *null set* or the *void set.*

E.g. Let A = $\{x : 1 < x < 2, x$ is a natural number$\}$. Then A is the empty set, because there is no natural number between 1 and 2.

(i) C = $\{x : x$ is an even prime number greater than 2$\}$.Then C is the empty set, because 2 is the only even prime number.

(ii) B = $\{x : x^2 - 2 = 0$ and x is rational number$\}$. Then B is the empty set because the equation $x^2 - 2 = 0$ is not satisfied by any rational value of x.

Finite and infinite sets -

- A set which is empty or consists of a definite number of elements is called *finite* otherwise, the set is called *infinite*.

Examples :

(i) Let W be the set of the days of the week. Then W is finite.

(ii) Let S be the set of solutions of the equation $x^2 - 16 = 0$. Then S is finite.

(iii) Let G be the set of points on a line. Then G is infinite.

Equal sets -

- Two sets A and B are said to be *equal* if they have exactly the same elements and we write A = B.
- Otherwise, the sets are said to be *unequal* and we write A ≠ B.

E.g. Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and also these are less than 6.

Subsets & Superset -

Subset:

- A set A is said to be a subset of a set B if every element of A is also an element of B.

E.g. A = set of all students in your class, B = set of all students in your school

All students of a class(set A) will be in set B (school).

- It is denoted as $A \subset B$
- Also every set A is subset of itself , $A \subset A$

Superset:

B is called *superset* of A.

E.g. The set Q of rational numbers is a subset of the set R of real numbers

we write $Q \subset R$.

Imp. – $N \subset Z \subset Q \subset R$

Singleton Set -

- If a set A has only one element, then it is called a *singleton set.*

Power set –

- The collection of all the subsets of a set A is called the *power set of A.*

E.g. Let us write down all the subsets of the set {1, 2}.

the set has total 4 subsets i.e. - φ, { 1 }, { 2 } and { 1, 2 }.

Hence power set of set A, P(A) = { φ,{ 1 }, { 2 }, { 1,2 } }.

Imp. -In general, if A is a set with $n(A) = m$, then it can be shown that

$$n[P(A)] = 2^m$$

Universal set -

- A set containing all sets of a problem under consideration is called *Universal set.*
- It is denoted by U.

E.g. While studying the system of numbers, we are interested in the set of natural numbers and its subsets such as the set of all prime numbers, the set of all even numbers, and so forth. Here the real numbers, R is the "*Universal Set*".

(i) For rolling a die U = {1,2,3,4,5,6,}

(ii) For tossing a coin U = {Head, tail}.

Open & Closed intervals -

Let $a,b \in R$ and $a<b$

- Open interval is denoted by $(a,b) = \{x : a < x < b\}$. Endpoint elements are NOT included.
- Closed interval is denoted by $[a,b] = \{x : a \leq x \leq b\}$. Endpoint elements are included.
- We can have open interval on one end and closed on other i.e.

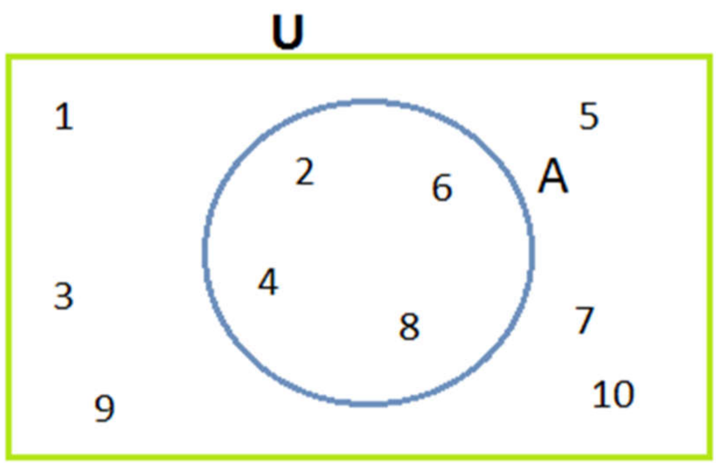
- $[a,b) = \{x : a \leq x < b\}$; an open interval from a to b, including a but excluding b.

- $(a,b] = \{x : a < x \leq b\}$; an open interval from a to b, excluding a but including b.

Venn diagram -

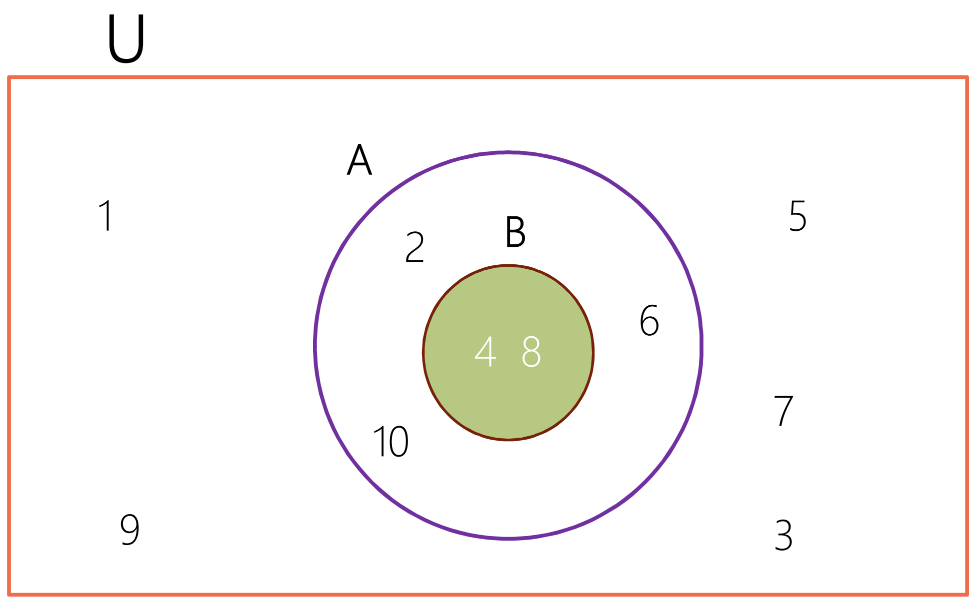
- Most of the relationships between sets can be represented by means of diagrams which are known as *Venn diagrams.*
- These diagrams consist of *rectangles* and closed curves usually *circles.*
- The universal set is represented usually by a rectangle and its subsets by circle.

E.g. In the fig. , U = {1,2,3, …, 10} is the universal set of which, A = {2,4,6,8} is a subset.



More e.g. -

- U = {1,2,3, ..., 10} is the universal set of which A = {2,4,6,8,10} and B = {4, 8} are subsets , and also B ⊂ A.



Operations on sets –

- Union of sets
- Intersection of set
- Difference of sets

Union of sets –

- Let A and B be any two sets.
- The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.
- The symbol '∪' is used to denote the *union*. *Symbolically, we write* A ∪ B *and usually read as 'A union B'.*

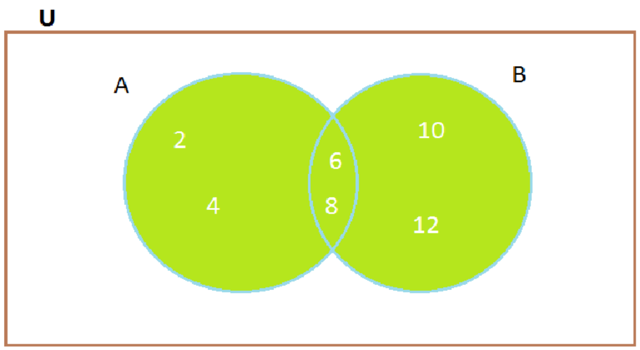
E.g. Let A = { 2, 4, 6, 8} and B = { 6, 8, 10, 12}.

Find A ∪ B.

Sol. - A ∪ B = { 2, 4, 6, 8, 10, 12}

Note that the common elements 6 and

8 have been taken only once while writing A ∪ B.



Properties of Union of sets -

(i) $A \cup B = B \cup A$ (Commutative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

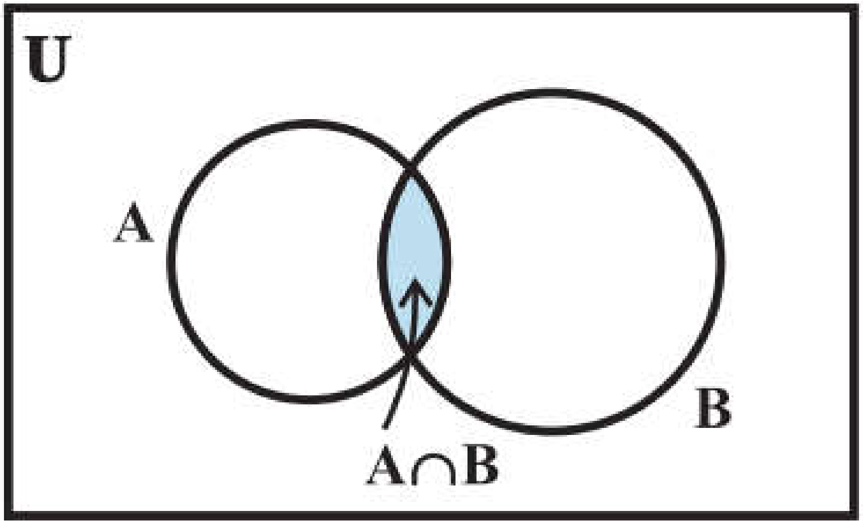
(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (Law of U)

Exercise – Q.1

Intersection of sets -

- The intersection of sets A and B is the set of all elements which are *common* to both A and B.
- The intersection of two sets A and B is the set of all those elements which belong to both A and B.
- The symbol '∩' is used to denote the *intersection*.



Some Properties of Operation of Intersection

(i) $A \cap B = B \cap A$ (Commutative law).

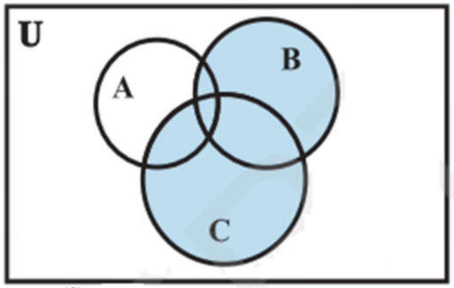
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U).

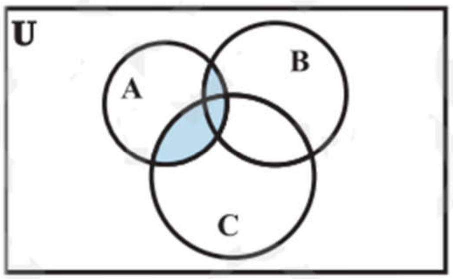
(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law)

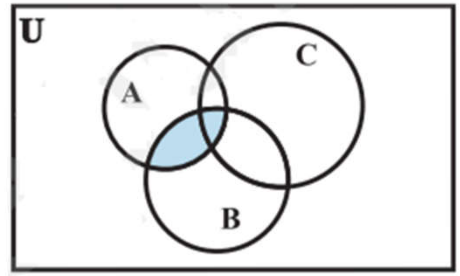
i.e.,$\cap$ distributes over $\cup$



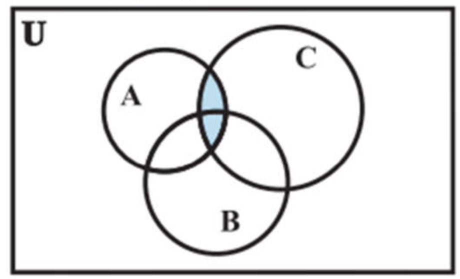
(i) $(B \cup C)$



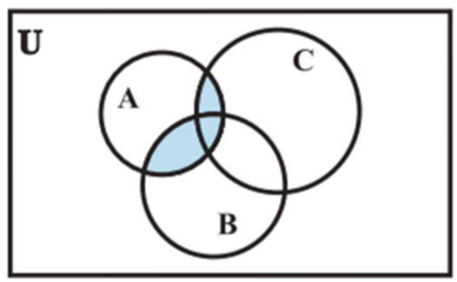
(ii) $A \cap (B \cup C)$



(iii) $(A \cap B)$



(iv) $(A \cap C)$



(v) $(A \cap B) \cup (A \cap C)$

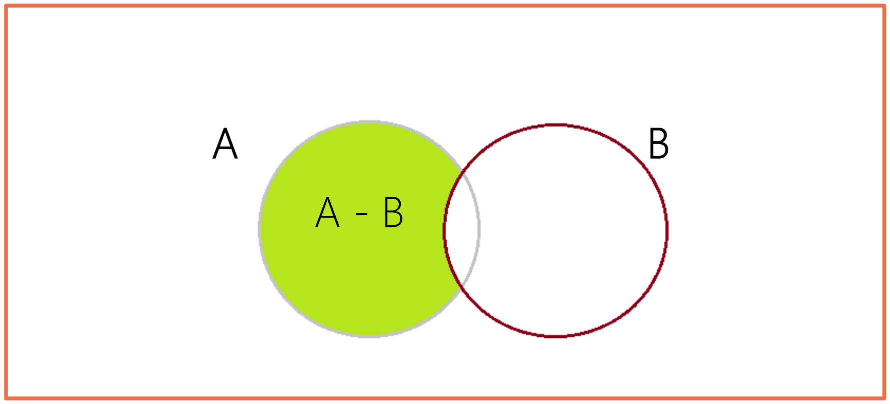
Difference of sets -

- The difference of the sets A and B in this order is the set of elements which belong to A but not to B.
- Symbolically, we write A – B and read as "A minus B"

E.g. Let A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }. Find A – B and B – A.

Solution A – B = { 1, 3, 5 }, since the elements 1, 3, 5 belong to A but not to B and B – A = { 8 }, since the element 8 belongs to B and not to A.

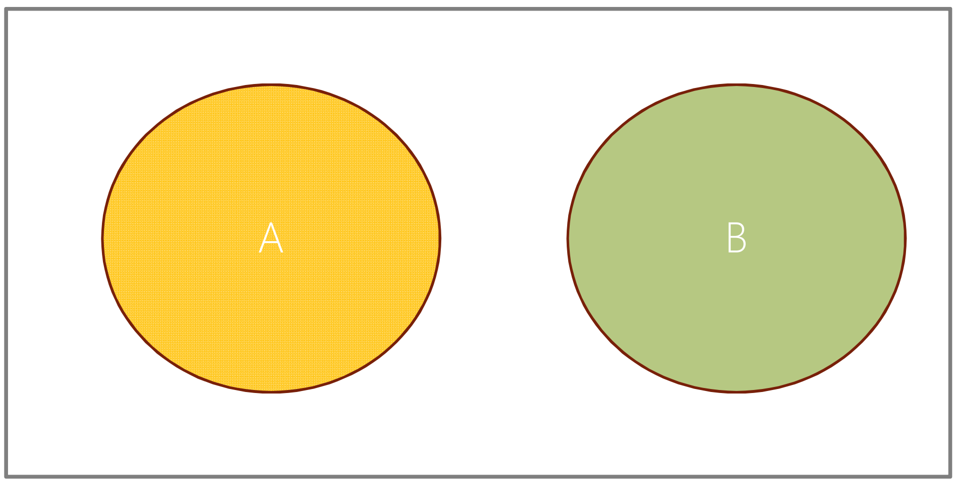
*Note- We note that A – B ≠ B – A.



Disjoint sets -

- If A and B are two sets such that $A \cap B = \phi$, then A and B are called *disjoint sets.*

U



Complement of a set -

- Let U be the universal set and 'A' a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A.
- Symbolically, we write A′ to denote the complement of A with respect to U. Thus $A' = \{x : x \in U$ and $x \notin A\}$.
- $A' = U - A$

E.g. U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 3, 5, 7, 9}. Find A′.
Solution We note that 2, 4, 6, 8, 10 are the only elements of U which do not belong to A. Hence A′ = { 2, 4, 6, 8,10 }.

E.g. Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.
Find A′, B′ , $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$

Properties of Complement of set -

1. Complement laws: (i) $A \cup A' = U$

(ii) $A \cap A' = \phi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

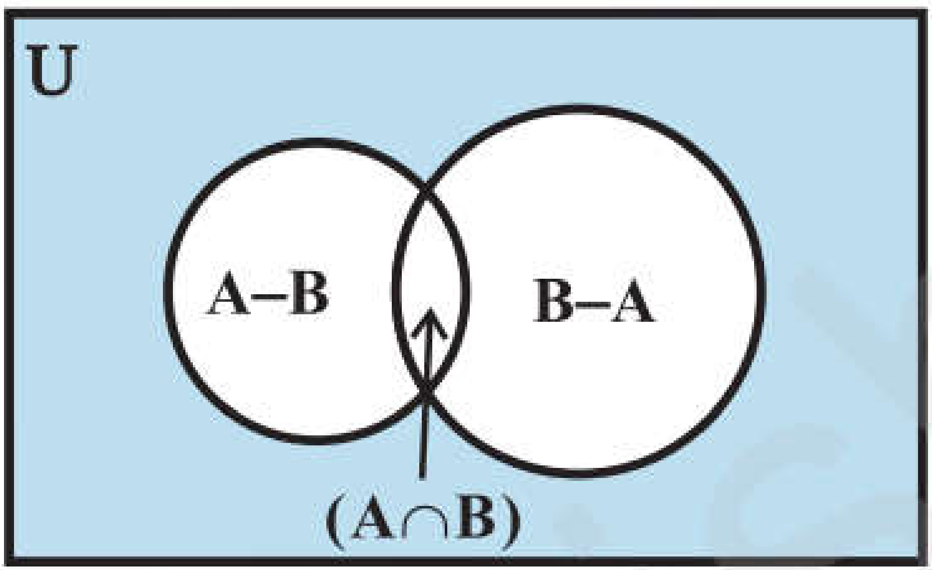
4. Laws of empty set and universal set $\phi' = U$ and $U' = \phi$.
These laws can be verified by using Venn diagrams

Exer.Q.1, Q.5

Number of Elements in set

If A, B, & C are finite sets, then

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$



- E.g. If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have ?
- E.g. 27,34

End ☺