10TH STD NOTES– REAL NUMBERS

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Chapter - Real Numbers

- Introduction
- Euclid's division Lemma
- Euclid's division algorithm
- Fundamental theorem of arithmetic and its applications
- Rational & Irrational number revisit.

Introduction

- Natural Numbers – N – [1, ∞)
  - Also can be called as Positive Integers
- Whole Numbers – W – [0, ∞).
  - Also can be called as non-negative integers
- Integers – Z - (-∞ , ∞)
  - Positive Integers - [1, ∞)
  - Negative Integers - (-∞, -1]
  - 0 is neither positive nor negative integer
- Rational Numbers – Q. Format is p/q where $p,q \in Z$ and $q \neq 0$
- Irrational Numbers – Q’ (Q prime) – R- Q or R \ Q, $P=\{x \mid x \in R \wedge x \notin Q\}$

Euclid’s division lemma

- Definition : *Given positive integers a and b, there exist unique integers q and r satisfying* $a = bq + r$, $0 \le r < b$.
- A lemma is a **proven statement** used for proving another statement.
- Euclid’s division algorithm is based on this lemma.

Euclid’s division lemma

- Examples :

  1st Example: 15 and 7.
- Considering 15 as dividend and 7 as divisor, then dividing 15 by 7, we get quotient as 2 and remainder as 1.
- So $15 = 7 * 2 + 1$ where $0 < 1 < 7$
- 2nd example 20 and 4, we get $20 = 4 * 5 + 0$ where $0 \leq 0 < 4$
- 3rd example : 3 and 5, if we consider 3 as dividend and 5 as divisor, then $3 = 5 * 0 + 3$ where $0 < 3 < 5$

Euclid’s Division Algorithm

- Euclid’s division algorithm is a technique to compute the Highest Common Factor(HCF) of two given positive integers.
- *To obtain the HCF of two positive integers, say c and d, with c >d, follow the steps below:*

Euclid’s Division Algorithm

- **Step 1** : Apply Euclid’s division lemma, to *c and d. So, we find whole numbers, q and r such that* $c = dq + r,\ 0 \le r < d$.
- ***Step 2*** *:* If $r = 0$, *d is the HCF of c and d. If* $r \ne 0$, *apply the division lemma to d and r.*
- ***Step 3*** *:* Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

Euclid’s Division Algorithm

- Example : 45, 117
  - Step 1 : 117 = 45 * 2 + 27 -- Remainder 27
  - Step 2 : 45 = 27 * 1 + 18 -- Remainder 18
  - Step 3 : 27 = 18 * 1 + 9 -- Remainder 9
  - Step 4 : 18 = 9 * 2 + 0 -- Remainder 0
  - So HCF is 9
- HCF (a, b) * LCM (a, b) = a * b

Fundamental Theorem of Arithmetic

- *Composite number : Has more than 2 factors*
- *Prime number : Has exactly 2 factors*
- *1 is neither a prime number nor a composite number*
- *Definition : Every composite number can be expressed (factorised) as a product of primes, and this factorisation is* ***unique****, apart from the order in which the prime factors occur.*

Revisiting rational numbers

- Real number is a rational number of the form, *p/q*
  - Case 1: where p and q are co-prime (HCF of p & q is 1)
  - Case 2 : p & q are not co-prime then divide them by common factor to convert them to ratio r/s where r & s are co-prime.
  - If the prime factorization of denominator (*q or s, as the case may be)*
    - $= 2^n * 5^m$, *where n, m are some non-negative integers. Then the rational number terminates.*
    - $\neq 2^n * 5^m$ then rational number does not terminate but recurring.

Revisiting Irrational numbers

- Rational * Irrational =Irrational where * means +, -, x, ÷
  - During division the denominator should not be zero.
- Prove that √3 is irrational
  - Solve such problems by first assuming contradictory statement.