

PROBABILITY

Life is a school of probability.

1. An integer is chosen at random from the first two hundreds digit. What is the probability that the integer chosen is divisible by 6 or 8. (Ans : $\frac{1}{4}$)

Ans: Multiples of 6 first 200 integers
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114,
120, 126, 132, 138, 144, 150, 156, 162, 168, 174, 180, 186, 192, 198

Multiples of 8 first 200 integers
8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, 128, 136, 144, 152, 160,
168, 176, 184, 192, 200

Number of Multiples of 6 or 8 = 50
 $P(\text{Multiples of 6 or 8}) = 50 / 200 = 1/4$

2. A box contains 12 balls out of which x are black .if one ball is drawn at random from the box what is the probability that it will be a black ball ? If 6 more black balls are put in the box ,the probability of drawing a black ball is now double of what it was before. Find x. (Ans: x = 3)

Ans: Random drawing of balls ensures equally likely outcomes

Total number of balls = 12

Total number pf possible outcomes = 12

Number of black balls = x

(1) out of total 12 outcomes, favourable outcomes = x

$$P(\text{black ball}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{x}{12}$$

Total number of possible outcomes

(2) if 6 more black balls are put in the bag, then

The total number of black balls = x + 6

Total number of balls in the bag = 12 + 6 = 18

According to the question

Probability of drawing black ball is second case

= 2 X probability drawing of black ball in first case

$$\frac{x+6}{18} = 2 \left(\frac{x}{12} \right)$$

$$\frac{x+6}{18} = \frac{x}{6}$$

$$6x + 36 = 18x$$

$$x = 3$$

hence number of black balls = 3

3. A bag contains 8 red balls and x blue balls, the odd against drawing a blue ball are 2: 5. What is the value of x? (Ans:20)

Ans: No. of blue balls be x
No. of red balls be 8
Total no. of balls = x + 8

$$\text{Probability of drawing blue balls} = \frac{x}{8+x}$$

$$\text{Probability of drawing red balls} = \frac{8}{8+x}$$

$$\frac{8}{8+x} : \frac{x}{8+x} = 2 : 5$$

$$2 \left(\frac{x}{8+x} \right) = 5 \left(\frac{8}{8+x} \right)$$

$$2x = 40$$

$$\therefore x = 20$$

4. A card is drawn from a well shuffled deck of cards
- What are the odds in favour of getting spade? (Ans: 1:3, 3:1, 3:10, 1:25)
 - What are the odds against getting a spade?
 - What are the odds in favour of getting a face card?
 - What are the odds in favour of getting a red king

Ans: Total cards 52
Spade = 13
Remaining cards 39

- i) The odds in favour of getting spade 13
The odds is not in favour of getting spade 39

$$= \frac{13}{52} : \frac{39}{52} = 1 : 3$$

ii) The odds against getting a spade 39
The odds not against getting a spade 13

$$= \frac{39}{52} : \frac{13}{52} = 3 : 1$$

iii) The odds in favour of getting a face card 12
The odds not in favour of getting a face card 40

$$= \frac{12}{52} : \frac{40}{52} = 3 : 10$$

iv) The odds in favour of getting a red king 2
The odds not in favour of getting a red king 50

$$= \frac{2}{52} : \frac{50}{52} = 1 : 25$$

5 A die is thrown repeatedly until a six comes up. What is the sample space for this experiment? HINT ;A= {6} B={1,2,3,4,5,}

Ans: The sample space is = {A, BA, BBA, BBBA, BBBBA.....}

6. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a foot ball match?

Ans: equally likely because they are mutually exclusive events .

7. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball , determine the number of blue balls in the bag. (Ans:10)

Ans: Let the number of blue balls in the bag be x

Then total number of balls in the bag = 5 + x

∴ Number of all possible outcomes = 5 + x

Number of outcomes favourable to the event of drawing a blue ball = x

(∵ there are x blue balls)

∴ Probability of drawing a blue ball $\frac{x}{5+x}$

Similarly, probability of drawing a red ball = $\frac{5}{5+x}$

According to the answer

$$\frac{x}{5+x} = 2 \left(\frac{5}{5+x} \right)$$

$$x = 10$$

8. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box the probability of drawing a black ball is now double of what it was before. Find x ? (Ans: 3)

Ans: Number of all possible outcomes = 12

Number of outcomes favourable to the event of drawing black ball = x

$$\text{Required probability} = \frac{x}{12}$$

Now when 6 more black balls are put in the box,

Number of all possible outcomes = $12 + 6 = 18$

Number of outcomes favourable to the event of drawing a black ball = $x + 6$

$$\therefore \text{Probability of drawing a black ball} = \frac{x+6}{18}$$

According to the question,

$$\frac{x+6}{18} = 2 \left(\frac{x}{12} \right)$$

$$\therefore x = 3$$

9. If 65% of the populations have black eyes, 25% have brown eyes and the remaining have blue eyes. What is the probability that a person selected at random has (i) Blue eyes (ii) Brown or black eyes (iii) Blue or black eyes

(iv) neither blue nor brown eyes (Ans: $\frac{1}{10}, \frac{9}{10}, \frac{3}{4}, \frac{13}{20}$)

Ans: No. of black eyes = 65
No. of Brown eyes = 25
No. of blue eyes = 10
Total no. of eyes = 180

i) $P(\text{Blue eyes}) = \frac{10}{180} = \frac{1}{18}$

ii) $P(\text{Brown or black eyes}) = \frac{90}{180} = \frac{1}{2}$

\therefore Favourable number of elementary events = 1

\therefore Probability of getting the card of queen = $\frac{1}{5}$

(ii) Now, the total number of elementary events = 4

(a) Since, there is only one ace

\therefore Favourable number of elementary events = 1

\therefore Probability of getting an ace card = $\frac{1}{4}$

(b) Since, there is no queen (as queen is put aside)

\therefore Favourable number of elementary events = 0

\therefore Probability of getting a queen = $\frac{0}{4} = 0$

14. A number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3. What is the probability that $|x| < 2$ (Ans: $\frac{3}{7}$)

Ans : x can take 7 values

To get $|x| < 2$ take -1, 0, 1

Probability ($|x| < 2$) = $\frac{3}{7}$

15. A number x is selected from the numbers 1,2,3 and then a second number y is randomly selected from the numbers 1,4,9. What is the probability that the product xy of the two numbers will be less than 9? (Ans: $\frac{5}{9}$)

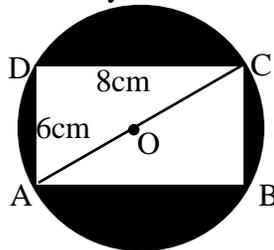
Ans : Number X can be selected in three ways and corresponding to each such way there are three ways of selecting number y . Therefore, two numbers can be selected in 9 ways as listed below:

(1,1), (1,4), (2,1), (2,4), (3,1)

\therefore Favourable number of elementary events = 5

Hence, required probability = $\frac{5}{9}$

16. In the adjoining figure a dart is thrown at the dart board and lands in the interior of the circle. What is the probability that the dart will land in the shaded region.



(Ans: $\frac{25\pi - 48}{25\pi}$)

Ans: We have

$$AB = CD = 8 \text{ and } AD = BC = 6$$

using Pythagoras Theorem in ΔABC , we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 8^2 + 6^2 = 100$$

$$AC = 10$$

$$OA = OC = 5 \quad [\because O \text{ is the midpoint of } AC]$$

$$\therefore \text{Area of the circle} = \pi (OA)^2 = 25\pi \text{ sq units } [\because \text{Area} = \pi r^2]$$

$$\text{Area of rectangle } ABCD = AB \times BC = 8 \times 6 = 48 \text{ sq units}$$

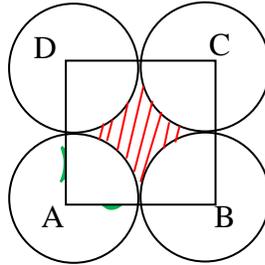
$$\text{Area of shaded region} = \text{Area of the circle} - \text{Area of rectangle } ABCD$$

$$\text{Area of shaded region} = 25\pi - 48 \text{ sq unit.}$$

Hence

$$P(\text{Dart lands in the shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of circle}} = \frac{25\pi - 48}{25\pi}$$

17. In the fig points A, B, C and D are the centres of four circles, each having a radius of 1 unit. If a point is chosen at random from the interior of a square ABCD, what is the probability that the point will be chosen from the shaded region.



$$(\text{Ans: } \frac{4 - \pi}{4})$$

Ans: Radius of the circle is 1 unit

$$\text{Area of the circle} = \text{Area of 4 sector}$$

$$\pi r^2 = \pi \times 1^2 = \pi$$

$$\text{Side of the square } ABCD = 2 \text{ units}$$

$$\text{Area of square} = 2 \times 2 = 4 \text{ units}$$

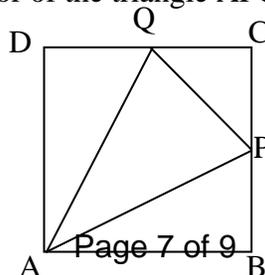
$$\text{Area shaded region is}$$

$$= \text{Area of square} - 4 \times \text{Area of sectors}$$

$$= 4 - \pi$$

$$\text{Probability} = \left(\frac{4 - \pi}{4} \right)$$

18. In the adjoining figure ABCD is a square with sides of length 6 units points P & Q are the mid points of the sides BC & CD respectively. If a point is selected at random from the interior of the square what is the probability that the point will be chosen from the interior of the triangle APQ.



$$\left(\text{Ans: } \frac{3}{8}\right)$$

Ans: Area of triangle PQC = $\frac{1}{2} \times 3 \times 3 = \frac{9}{2} = 4.5$ units

Area of triangle ABP = $\frac{1}{2} \times 6 \times 3 = 9$

Area of triangle ADQ = $\frac{1}{2} \times 6 \times 3 = 9$

Area of triangle APQ = Area of a square – (Area of a triangle PQC + Area of triangle

ABP + Area of triangle ADQ)

$$= 36 - (9 + 4.5)$$

$$= 36 - 13.5$$

$$= 22.5$$

Probability that the point will be chosen from the interior of the triangle APQ = $\frac{22.5}{36}$

$$= \frac{22.5}{36} = \frac{5}{8}$$

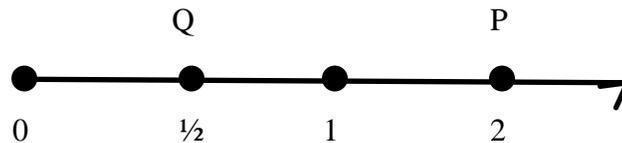
19. In a musical chair game the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the half minute after starting.

$$\left(\text{Ans: } \frac{1}{4}\right)$$

Ans: Here the possible outcomes are all the numbers between 0 and 2.

This is the portion of the number line from 0 to 2 as shown in figure.

Let A be the event that ‘the music is stopped within the first half minute.’ Then, outcomes favorable to event A are all points on the number line from O to Q i.e., from 0 to 1/2 .



The total number of outcomes are the points on the number line from O to P i.e., 0 to 2.

$$\therefore P(A) = \frac{\text{Length of OQ}}{\text{Length of OP}} = \frac{1/2}{2} = \frac{1}{4}$$

$$\frac{1}{4}$$

20. A jar contains 54 marbles each of which is blue , green or white. The probability of selecting a blue marble at random from the jar is $\frac{1}{3}$ and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain? (Ans:12)

Ans: Let there be b blue, g green and w white marbles in the marbles in the jar. Then,

$$b + g + w = 54$$

$$\therefore P(\text{Selecting a blue marble}) = \frac{b}{54}$$

It is given that the probability of selecting a blue marble is $\frac{1}{3}$.

$$\therefore \frac{1}{3} = \frac{b}{54} \Rightarrow b = 18$$

We have,

$$P(\text{Selecting a green marble}) = \frac{4}{9}$$

$$\Rightarrow \frac{g}{54} = \frac{4}{9} \quad [\because P(\text{Selecting a green marble}) = \frac{4}{9} \text{ (Given)}]$$

$$\Rightarrow g = 24$$

Substituting the values of b and g in (i), we get

$$18 + 24 + w = 54 \Rightarrow w = 12$$