VECTORS

In applied mathematics, Physics and mechanics, there exists two types of physical quantities called *scalars* and *vectors*. Those quantities which involve only magnitudes and no direction are called *scalars*. Those quantities which involve both magnitude and direction are called *vectors*.

EXAMPLE:

Distance, Area, Volume, Mass, Speed, Temperature are *scalars* and Velocity, Acceleration, Force, Electric field etc are *vectors*.

1.1 REPRESENTATION OF A VECTOR:

Vector quantity required length. In a precise manner, vector means " A directed line segment **"** . It is represented by a line OP directed from an initial point O to the terminal point P and denoted by $\overrightarrow{OP}$. Hence the length of the vector $\overrightarrow{OP}$ denoted by

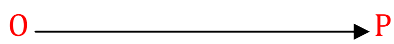


Figure (1)

$|\overrightarrow{OP}|$ **=** OP is called magnitude or modulus of the vector . The arrow mark in Figure (1)

$\overrightarrow{OP}$ denoted direction if a = op, it represents length then $\vec{a} = \overrightarrow{op}$, represents vector. Arrow mark on the head of a quantity represents a vector .

1.2 KINDS OF VECTORS:

1. NULL VECTORS:

   A vector having initial and terminal point co – incident is called as null (or zero) vector . In other wards a vector with zero magnitude is called Null Vector.

2. UNIT VECTOR:

A vector having its modulus as unity is called unit vector . If $\vec{a}$ is any vector and $|\vec{a}| = 1$, then $\vec{a}$ is called Unit Vector is denoted by

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

3. EQUAL VECTORS:

Two vector having same magnitude and same direction are called Equal Vectors .

Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal if

$$|\vec{a}| = |\vec{b}|$$

4. EQUAL VECTORS:

If a is any given vector, then $-\vec{a}$ is called Negative of $\vec{a} \cdot \vec{a}$ and $-\vec{a}$ and have same magnitude but opposite direction .

5. SCALAR MULTIPLE OF A VECTORS:

If $\vec{a}$ is any vector and λ is any scalar, then $\lambda\vec{a}$ called *Scalar multiple* of $\vec{a}$.

Scalar multiple of a vector $\vec{a}$ is a vector whose magnitude is scalar times the magnitude of $\vec{a}$.

If $|\vec{a}| = 2$, $|2\vec{a}| = 2 \text{ X } 2 = 4$, $|\frac{1}{4}\vec{a}| = \frac{1}{4} \text{ X } 2 = \frac{1}{2}$

6. COLLINEAR VECTORS:

Two vectors parallel to the same line regardless of their magnitude and direction are called collinear. If $\vec{a}$ and $\vec{b}$ are any two collinear vectors, there exist λ (a scalar) such that $\vec{b} = \lambda\vec{a}$.

i.e., one of the two collinear vectors can be expressed as scalar multiple of the other .

7. COPLANAR VECTORS:

A system of vectors lying in the parallel planes or lie in the same plane are said to be Coplanar Vector .

Evidently any two vectors always are Coplanar.

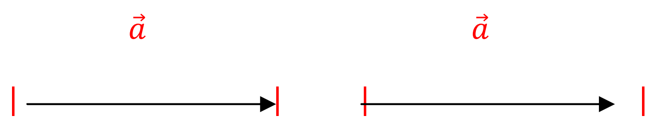


Figure (2)

8. LIKE AND UNLIKE VECTORS:

Two vectors, which are collinear and have same direction are called Like Vecrors .

The vectors whose direction are opposite are called Unlike Vectors .

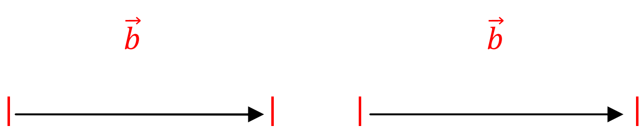


Figure (3)

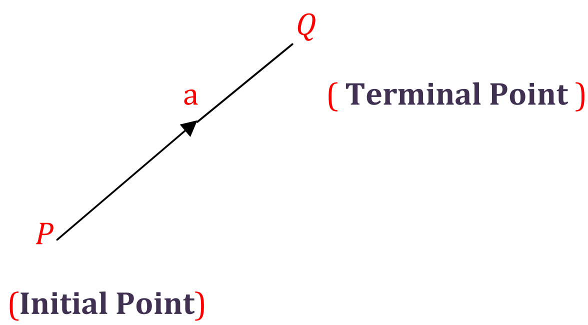
1.3 POSITION VECTOR OF A POINT (P.V. OF A POINT):

If A,B,C are any three points in a plane and O is fixed point in the plane. Then $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ are called position vectors of points A,B,C respectively with reference to O.The position vector of a point specifies the position of a point relative to an arbitrary chosen point .

1.4 VECTOR NOTATION:

- **A scalar quantity or scalar is a quantity that possesses only magnitude.**
- **A vector quantity is a quantity that possesses both magnitude and direction.**

- **A vector is represented by a directed line segment. The arrow on the line segment indicates the direction of the vector. The length segment represents the magnitude of the vector.**



The vector shown above is denoted by $\overrightarrow{PQ}$ or a in written work, we use $\underline{a}$ or $\tilde{a}$.

- **The magnitude of $\overrightarrow{PQ}$ is denoted by | $\overrightarrow{PQ}$ |, | a | or | $\tilde{a}$ |.**

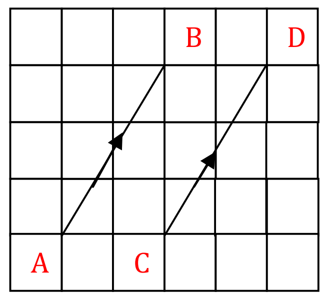
| $\overrightarrow{PQ}$ | = Length of $\overrightarrow{PQ}$

EQUAL VECTORS:

Two vectors are equal if they have the same magnitude and the same direction.

e.g. $\overrightarrow{AC} = \overrightarrow{CD}$

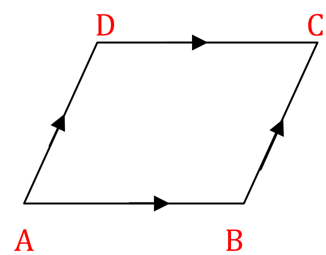
$\Rightarrow |\overrightarrow{AB}| = |\overrightarrow{CD}|$ and $AB \parallel CD$



e.g. If ABCD is a parallelogram, then the opposite sides are equal vectors .

$\overrightarrow{AC} = \overrightarrow{CD}$

$\overrightarrow{AD} = \overrightarrow{BC}$



NEGATIVE VECTORS:

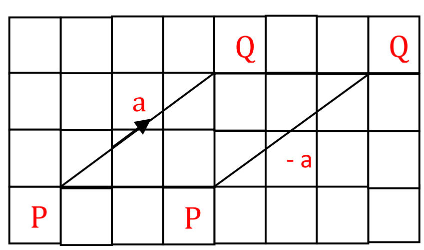
The vectors are negative vectors of each other if they have the same magnitude but are in opposite direction.

$\overrightarrow{PQ} = \overrightarrow{QP}$ are negative vectors of each other.

$\overrightarrow{PQ} = -\overrightarrow{QP}$ or

$\overrightarrow{QP} = -PQ$ or

a = - (- a)



IMPORTANT: $|\overrightarrow{PQ}| = |\overrightarrow{QP}|$

ZEROVECTOR:

- Given any vector **'a'**, there exists a negative vector **'–a'**, such that

a + (- a) = (- a) + a = 0

0 is called the zero vector and is written as Q.

- **The zero vector or null vector is a vector with zero magnitude and no direction.**

IMPORTANT: $|\overrightarrow{PQ}| + |\overrightarrow{QP}| = 0$

1.5 ADDITION OF VECTORS:

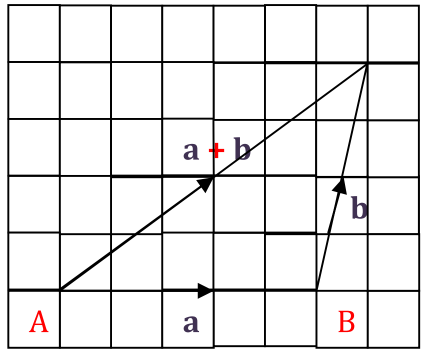
To add vectors, use the triangle law of vector addition or the parallelogram law of vector addition.

TRIANGLE LAW OF VECTOR ADDITION:

If two vectors a and b are represented by the sides $\overrightarrow{AB}$ *and* $\overrightarrow{BC}$ of a triangle, a+b is represented by the third side $\overrightarrow{AC}$.

$\overrightarrow{AB} + \overrightarrow{BC}$ $\overrightarrow{AC}$ or

$\overrightarrow{AC}$ = a + b



TIPS FOR STUDENTS:

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ ← Add head to tail for each pair of vectors

Must be the same point

Head   Tail   Head   tail

$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$ ← Add head to tail for each pair of vectors

Same point

Head   Tail   Head   tail

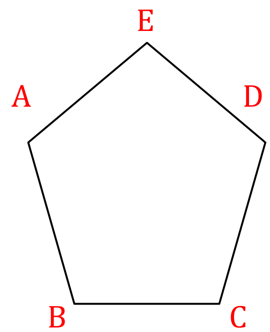
e.g. ABCDE **is a regular pentagon. Simplify each of the following.**

a) $\overrightarrow{AB} + \overrightarrow{BE} + \overrightarrow{EC}$

$= (\overrightarrow{AB} + \overrightarrow{BE}) + \overrightarrow{EC}$ (associate law)

$= \overrightarrow{AE} + \overrightarrow{EC}$ (Triangle law)

$= \overrightarrow{AC}$ (Triangle law)



b) $\overrightarrow{CB} - \overrightarrow{AB} + \overrightarrow{AE} - \overrightarrow{DE}$

$= (\overrightarrow{CB} + \overrightarrow{BA}) + (\overrightarrow{AE} + \overrightarrow{ED})$

$= (\overrightarrow{CA} + \overrightarrow{AD})$ (Triangle law)

$= \overrightarrow{CD}$ (Triangle law)

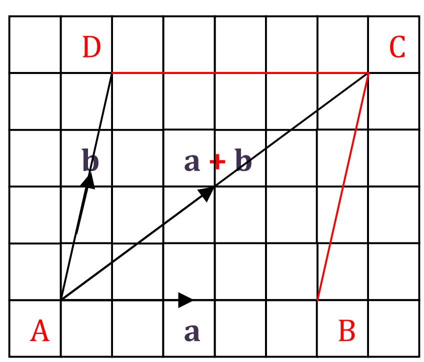
$- \boldsymbol{AB} = \overrightarrow{\boldsymbol{BA}}$
$- \overrightarrow{\boldsymbol{DE}} = \overrightarrow{\boldsymbol{ED}}$

PARALLELOGRAM LAW OF VECTOR ADDITION:

- If two vectors a and b are represented by the adjacent sides $\overrightarrow{AB}$ *and* $\overrightarrow{AD}$ of a parallelogram, then a = b is represented by the diagonal of the parallelogram $\overrightarrow{AC}$.

$\overrightarrow{\boldsymbol{AB}} + \overrightarrow{\boldsymbol{AD}} = \overrightarrow{\boldsymbol{AC}}$

$\overrightarrow{\boldsymbol{AC}} = \mathbf{a + b}$



e.g.- PQRS is a parallelogram .T is the intersection of its diagonals. Simplify each of the following.

a) $\overrightarrow{QP} + \overrightarrow{QR} = \overrightarrow{QS}$ (Parallelogram law)

b) $\overrightarrow{PQ} + \overrightarrow{PS} + \overrightarrow{RQ}$

$= (\overrightarrow{PQ} + \overrightarrow{PS}) + \overrightarrow{RQ}$ (Associtative law)

$= \overrightarrow{PR} + \overrightarrow{RQ}$ (Parallelogram law)

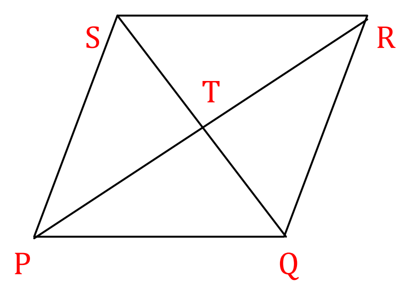
$= \overrightarrow{PQ}$ (Triangle law)

Parallelogram law **Triangle law**:

c) $\overrightarrow{PQ} + \overrightarrow{PS} + \overrightarrow{RS} + \overrightarrow{ST}$

$= \overrightarrow{PR} + \overrightarrow{RT}$

$= \overrightarrow{PT}$ (**Triangle law**)



- **Vector addition obeys the following rules.**

If a , b and c are vectors, then

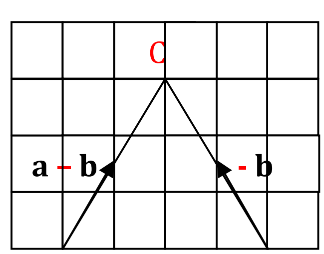
✓ **a + b = b + a (Commutative law)**

✓ **(a + b) + c = a + (b + c) (Associative law)**

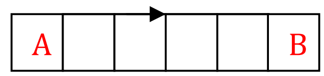
1.6 SUBTRACTION OF VECTORS:

To subtract two vectors, add the first vector to the negative of the second vector.

a − b = a + (- b)



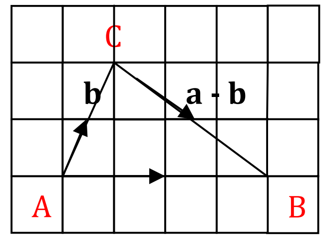
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ (Triangle law)



= a + (- b)

= a – b

$\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$ (**Triangle law**)



b + $\overrightarrow{CB}$ = **a**

$\overrightarrow{CB}$ = **a – b**

TIPS FOR STUDENTS:

$\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$ (**Triangle law of addition**)

$\overrightarrow{CB} = \overrightarrow{AB}$

$\therefore \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$

End

Start

Start

End

Must be the same point

$\therefore \overrightarrow{PQ} - \overrightarrow{PR} = \overrightarrow{RQ}$

End

Start

Start

End

Same Point

EXAMPLE:

1. If $\vec{a}$ is a vector of magnitude 4 units, what are $2\vec{a}$, $\sqrt{2}.\vec{a}$, $\frac{1}{4}\ \vec{a}$

SOLUTION:

Magnitude of $2\vec{a} = |2.\vec{a}| = 2|\vec{a}| = 2 \times 4 = 8$

Magnitude of $\sqrt{2}.\vec{a} = |\sqrt{2}.\vec{a}| = \sqrt{2}\,|\vec{a}\,| = 2\sqrt{2}$

Magnitude of $\frac{1}{4}\ .\vec{a} = |\frac{1}{4}\ .\vec{a}| = \frac{1}{4}\ |\vec{a}| = \frac{1}{4} X\ 4 = 1$

2. If $\vec{a}$ is vector of magnitude 5 unit v ector in the direction of $\vec{a}$.

SOLUTION:

$|\ \vec{a}\ | = 5$ (Given)

Unit vector, in the direction of a $\vec{a} = \hat{a} = \frac{\vec{a}}{|\ \vec{a}\ |} = \frac{\vec{a}}{5}$

1.7 ADDITION OF VECTORS:

Addition of vectors is given by well known law of parallelogram of forces. It states that " **If two vectors acting at a point are represented by the adjacent sides of a parallelogram, then their resultant is given by the diagonal of the parallelogram passing through that point** ".

Two forces acting at a point O can be represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ as adjacent sides and diagonal $\overrightarrow{OC}$represents resultant as diagonal of the parallelogram OACB .

i.e., $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$ Or $\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$

The same geometrical construction is used in defining vector addition.

If Δ ABC is any triangle then,

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

Figure (2)), similarly, $\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$

In addition of vectors, initial point of the second vector coincides with terminal point of the first vector. Then sum is obtained by joining initial point of the first to terminal point of the second vector.

1.8 SUBTRACTION OF VECTORS:

Since $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, $\overrightarrow{AB} - \overrightarrow{AC} = -\overrightarrow{BC}$

$$\therefore \quad \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

In subtraction of vectors,the initial point of two vectors coincide.

$\overrightarrow{OP} - \overrightarrow{OQ} = \overrightarrow{QP}$, $\overrightarrow{OM} - \overrightarrow{OL} = \overrightarrow{LM}$

Addition of vectors: $\vec{a}$, $\vec{b}$, $\vec{c}$, satisfies the following basic laws.

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
3. $(m + n)\vec{a} = m\vec{a} + n\vec{a}$
4. $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$
5. $m(n\vec{a}) = mn\vec{a}$
6. $\vec{a} + \vec{o} = \vec{o} + \vec{a} = \vec{a}$

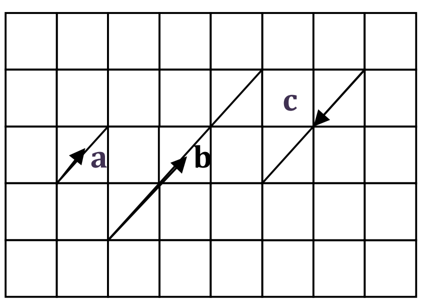
1.9 SCALAR MULTIPLICATION OF VECTOR:

- When a vector **a** is multiplied by a scalar **k**, the resulting vector **ka** has a magnitude **k** times of **a**, $|ka| = k\,|a|$.

ka is parallel to **a** and is in

- ✓ **The same direction as a if k is positive (k > 0).**
- ✓ **The opposote direction as a if k is positive (k < 0).**

e.g.-



b = 3a

c = -2a

- **Vector a is parallel to vector b if and only if**

a = kb

- **Scalar multiplication of vectors obey the following rules.**

If a and b are vectors, and m and n are real numbers, then

- ✓ m (na) = n (ma) = (mn) a
- ✓ (m + n) a = ma + na
- ✓ m (a + b) = ma + mb

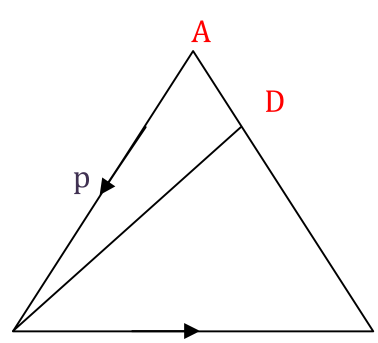
- If the points A,B and C lie in a straight (i.e. they are collinear), then

$$\overrightarrow{AB} = k\ \overrightarrow{BC}.$$

EXAMPLE:

In the diagram, AD = $\frac{1}{4}$ AC , $\overrightarrow{AB}$ = **p** and $\overrightarrow{BC}$ = **q** .

FIND :



a) $\overrightarrow{AC}$,

b) $\overrightarrow{DC}$,

c) $\overrightarrow{BD}$.

B q C

SOLUTION:

a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \mathbf{p} + \mathbf{q}$

b) $\overrightarrow{DC} = \frac{3}{4}\overrightarrow{AC}$

$= \frac{3}{4}(\mathbf{p} + \mathbf{q})$

$= \frac{3}{4}\mathbf{p} + \frac{3}{4}\mathbf{q}$

Given $AD = \frac{1}{4} AC$

$\therefore \quad \overrightarrow{DC} = \frac{3}{4}\overrightarrow{AC}$

c) $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ **(Triangle law)**

$= \overrightarrow{BC} + (-\overrightarrow{DC})$

$= \mathbf{q} - \left(\frac{3}{4}\mathbf{p} + \frac{3}{4}\mathbf{q}\right)$

$= \mathbf{q} - \frac{3}{4}\mathbf{p} - \frac{3}{4}\mathbf{q}$

$= \frac{1}{4}\mathbf{q} - \frac{3}{4}\mathbf{p}$

1.10 COMPONENTS OF A VECTOR IN 2 MUTUALLY PERPENDICULAR DIRECTIONS (i.e.,IN PLANE):

It is convention that $\vec{i}$, $\vec{j}$ are taken as unit vectors in the positive directions of x and y axis. Let P(x, y) be any point. Then, $\overrightarrow{OP}$ is called the position vector of the point P.

Draw PM ⊥ x - axis. Join OP. Now OM = x and unit vector in the direction of

OM is $\hat{\imath}$.

$\therefore$ **$\overrightarrow{OM}$ = x$\hat{\imath}$**

MP = y and unit vector in the direction of y is $\hat{\jmath}$, $\therefore$ $\overrightarrow{MP}$ = y$\hat{\jmath}$

MP = y and unit vector in the direction of y is $\hat{\jmath}$, $\therefore$ $\overrightarrow{MP}$ = y$\hat{\jmath}$

Now, $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = x\hat{\imath} + y\hat{\jmath}$

i.e., the position vector of the point p(x, y) is

$$\vec{r} = \overrightarrow{OP} = x\hat{\imath} + y\hat{\jmath}$$

The length of $\overrightarrow{OP} = |\vec{r}| = \sqrt{OM^2 + MP^2}$

$\therefore |x\hat{\imath} + y\hat{\jmath}| = \sqrt{x^2 + y^2}$

The direction of $\overrightarrow{OP}$ with x – axis is given by

$$\tan\theta = \frac{y}{x} \quad \text{or} \quad \theta = \tan^{-1}\frac{y}{x}$$

Examples:

1. The position vector (p. v.) of a point A (3, 5) is

   $\overrightarrow{OA} = 3\hat{\imath} + 5\hat{\jmath}$

2. The position vector of the point p (3,-4) is $\overrightarrow{OP} = 3\hat{\imath} - 4\hat{\jmath}$ and magnitude is $|3\hat{\imath} - 4\hat{\jmath}| = \sqrt{3^2 + (-4)^2} = 5$
3. If A = (2, 3), B = (-4, 5), find the position vector of A and B and also find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$

Solution:

$\overrightarrow{OA} = 2\hat{\imath} - 3\hat{\jmath}$, $\overrightarrow{OB} = -4\hat{\imath} + 5\hat{\jmath}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-4\hat{\imath} + 5\hat{\jmath}) - (2\hat{\imath} - 3\hat{\jmath})$

$\overrightarrow{AB}$ = $|-6\hat{\imath} + 8\hat{\jmath}|$ = $\sqrt{(-6)^2 + 8^2}$ = 10

1.11 RIGHT HANDED RECTANGULAR SYSTEM:

The three mutually perpendiculars lines XOX', YOY' and ZOZ' meet at O. the system of axes so choosen from a right handed co-ordinate system such that ox is turned towards OY about OZ through a small angle a right handed screw would advance along the positive direction OZ.

$\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are unit vectors along $\overrightarrow{OX}$, $\overrightarrow{OY}$ and $\overrightarrow{OZ}$. Let P = (x, y, z) be any point in space.The position vector of the point P (x, y, z) is

$\overrightarrow{OP}$ = $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Its magnitude is

$\overrightarrow{OP}$ = $\sqrt{x^2 + y^2 + z^2}$

If $\overrightarrow{OP}$ makes an angle α, β *and* γ with positive directions of x, y and z axis, then $\cos\alpha$ $\cos\beta$ and $\cos\gamma$ are the direction cosines of $\overrightarrow{OP}$

Examples:

1. The position vector of the point p (2, 3, -1) is $2\hat{\imath} + 3\hat{\jmath} - 1\hat{k}$

   Find its magnitude.

   Solution:

   magnitude is $|2\hat{\imath} + 3\hat{\jmath} - 1\hat{k}| = \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$

2. If A and B are the points A (-1, 3, 2) and B (3, 0 4). Find $\overrightarrow{AB}$.

   Solution:

   $\overrightarrow{AB}$ = $\overrightarrow{OB}$ - $\overrightarrow{OA}$

   Where $\overrightarrow{OB} = 3\hat{\imath} + 0\hat{\jmath} - 4\hat{k}$ $\overrightarrow{OA} = -\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$\therefore \quad \overrightarrow{AB} = (3\hat{\imath} + 0\hat{\jmath} - 4\hat{k}) - (-\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

$\therefore \quad \overrightarrow{AB} = 4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$\therefore \quad |\overrightarrow{AB}| = \sqrt{(4)^2 + (-3)^2 + (2)^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

3. Prove the following by vector method.

i) If a pair of opposite sides of a quadrilateral are equal and parallel, they form parallelogram.

ii) The diagonals of a parallelogram bisect each other.

Solution:

i) ABCD is a quadrilateral in which

$\overrightarrow{AB} = \overrightarrow{DC}$

(AB is parallel and equal to DC)

Now, $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= \overrightarrow{AB} + (\overrightarrow{BC} + \overrightarrow{BD})$

$= \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{DC}$ $\quad (\because \overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{BD})$

$= \overrightarrow{BC}$ $\quad (\because \overrightarrow{AB} = \overrightarrow{DC})$

That is AD is parallel and equal to BC. Therefore ABCD is a parallelogram.

ii) E is the point of intersection of diagonals AC and BD.

Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be position vectors of A, B, C, D respectively.

Position vector of AC is $\frac{\vec{a}+\vec{c}}{2}$

Position vector of BD is $\frac{\vec{b}+\vec{d}}{2}$

Since ABCD is a parallelogram $\overrightarrow{AB} = \overrightarrow{DC}$

$$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OD}$$

$$\therefore \vec{b} - \vec{a} = \vec{c} - \vec{d}$$

$$\vec{b} + \vec{d} = \vec{a} + \vec{c}$$

$$\frac{\vec{b}+\vec{d}}{2} = \frac{\vec{a}+\vec{c}}{2}$$

position vector of midpoint of AC = position vector of midpoint of BD.
Therefore, diagonal bisect each other.

1.12 PRODUCT OF TWO VECTORS:

There are two kinds of products,one is called scalar or dot product

and other vector or cross product.The scalar or dot product of two vectors a and b is denoted by $\vec{a}.\vec{b}$ or $\vec{a}\,\vec{b}$ and their vector or cross product is $\vec{a} \times \vec{b}$ or $[\vec{a}\,\vec{b}]$

1.12.(a) Definition:

Dot product: Let $\vec{a}$ = (a_1, a_2, a_3), $\vec{b}$ = (b_1, b_2, b_3) be two vectors. then,dot product of $\vec{a}$ and $\vec{b}$ is defined by $a_1b_1 + a_2b_2 + a_3b_3$.

$$\therefore \vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{a}.\vec{b} = \vec{b}.\vec{a}$$ (dot product is commutative)

b.) Vector product:

Let $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$

Then, the cross product of $\vec{a}$ and $\vec{b}$ is

$\vec{a} \times \vec{b} = \hat{\imath}\,(a_2b_3 - a_3b_2) - \hat{\jmath}\,(a_1b_3 - a_3b_1) + \hat{k}\,(a_1b_2 - a_2b_1)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \qquad [\textit{since}, \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}]$$

Vector product is not commutative.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = - \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = -\vec{b} \times \vec{a}$$

Example:

1. If $\vec{a} = (3, 2, 1)$, $\vec{b} = (2, 1, -1)$ find i) $\vec{a}.\vec{b}$ and ii) $\vec{a} \times \vec{b}$

Solution:

i.) $\vec{a}.\vec{b} = (3,2,1).(2,1,-1) = 3(2) + 2(1) + 1(-1) = 6 + 2 - 1 = 7$

ii.) $$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \vec{\imath}\,(-2-1) - \vec{\jmath}(-3-2) + \vec{k}\,(3-4) = -3\vec{\imath} + 5\vec{\jmath} - \vec{k}$$

1.13.) GEOMETRICAL MEANING OF SCALAR AND VECTOR PRODUCTS:

Let $\vec{a}$ and $\vec{b}$ be any two non-zero vectors.The dot product of $\vec{a}$ and $\vec{b}$ is equal to product of magnitude of $\vec{a}$ and $\vec{b}$ and cosine of the angle between them.

i.e., $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|.\cos\theta$

Also $\vec{a}.\vec{b} = |\vec{a}|$ x projection of $\vec{b}$ in the direction of $\vec{a}$.

1. If $\theta = 90^0$, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|.\cos 90^0$

   $\vec{a}.\vec{b} = 0$

   i.e.,vectors are perpendiculars to each other. Perpendicular vectors are also called orthogonal vectors.

2. If $\vec{a}$ is parallel to $\vec{b}$ (or coincident with $\vec{b}$), then $\theta = 0$

   $\therefore\ \vec{a}.\vec{b} = |\vec{a}||\vec{b}|$ $\vec{a}.\vec{a} = |\vec{a}|^2$

3. For unit vectors, $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$, $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$. Since $\hat{\imath}, \hat{\jmath}, \hat{k}$ are mutually perpendicular to each other.

   $\hat{\imath}.\hat{\imath} = |\hat{\imath}|^2 = 1$ similarly, $\hat{\jmath}.\hat{\jmath} = 1$, $\hat{k}.\hat{k} = 1$

1.14.) GEOMETRICAL MEANING OF $\vec{a}$ x $\vec{b}$.

The vector product of two non-zero vector $\vec{a}$ and $\vec{b}$ is given by $\vec{a}$ x $\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where θ is angle between them and $\hat{n}$ is a unit vector in the direction of $\vec{a}$ x $\vec{b}$.

i.e. $\vec{a}$ x $\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$

Where $\vec{a}$, $\vec{b}$ and $\vec{a}$ x $\vec{b}$ form a right handed system i.e.$\hat{n}$ is the unit vector, perpendicular to both $\vec{a}$ and $\vec{b}$.

$\therefore \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta|\hat{n}| = |\vec{a}||\vec{b}| \sin\theta \quad (|\hat{n}| = 1)$

= OA.OB.sinθ

= OA x perpendicular from B to A

= Area of parallelogram OACB

Thus, $\vec{a} \times \vec{b}$ is a vector, whose modulus gives the area of parallelogram OACB.

IMPORTANT FOR STUDENTS:

a) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (commutative law does not hold)

b) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

c) $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}| = |\vec{a}||\vec{b}|. \sin\theta$

d) If $\theta = 0$, $\sin\theta = 0$, $\therefore \vec{a} \times \vec{b} = 0$, i.e. $\therefore \vec{a} \times \vec{a} = 0$ i.e. vector are co-incident.

e) If $\vec{a}$ is perpendicular to $\vec{b}$, then $\theta = 90^0$, $\therefore$ $\sin\theta = \sin 90^0 = 1$

$$\therefore |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$$

f) For unit vector : i,j,k, i×i = j×j = k×k = 0 and ixj=k, jxk=i, kxi=j

1.15 AREA OF A TRIANGLE GIVEN TWO OF ITS SIDES $\vec{a}$ AND $\vec{b}$:

Let $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AC} = \vec{b}$ and $\lfloor BAC = \theta$ (radians)

Area of ABC = ½ x AB x CD

= ½ XAB.AC sinθ

= ½ $|\overrightarrow{AB}||\overrightarrow{AC}|\sin\theta$

= ½ $|\vec{a}||\vec{b}|.\sin\theta$

Area of Δ ABC = ½$|\vec{a} \times \vec{b}|$, where $\vec{a}$ and $\vec{b}$ are two sides.

1.16. AREA OF PARALLELOGRAM:

Area of parallelogram = AB X DE

$$= |\vec{a}||\vec{b}|.\sin\theta = |\vec{a} \times \vec{b}|$$

Thus, $|\vec{a} \times \vec{b}|$ represents area of parallelogram with $\vec{a}$ and $\vec{b}$ as adjacent sides.

1.17. PROJECTION OFONE VECTOR ON ANOTHER:

$\overrightarrow{LM}$ and $\overrightarrow{CD}$ are two vectors. Let θ be the angle between them.

Here, CA and DB ⊥ LM. Then AB is called the projection of $\overrightarrow{CD}$ on $\overrightarrow{LM}$.

$$Take\ \overrightarrow{LM} = \vec{a} \quad and \quad \overrightarrow{CD} = \vec{b}$$

Projection of $\overrightarrow{CD}$ on $\overrightarrow{LM}$

$$= AB = CE = CD\cos\theta$$

$$= |\overrightarrow{CD}|.\cos\theta = \frac{|\vec{b}|.|\vec{a}|.cos\theta}{|\vec{a}|}$$ (divide and multiply by $|\vec{a}|$.)

Projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

And projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Example1:

Show that the points whose position vectors are $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ form right angled triangle.

Solution:

Let $\overrightarrow{OA} = 3\hat{\imath} - 4\hat{\jmath} + \hat{k}$; $\overrightarrow{OB} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$; $\overrightarrow{OC} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$,

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$; $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\therefore \overrightarrow{AB} = \sqrt{(-1)^2 + 3^2 + (-5)^2} = \sqrt{35}$

$\overrightarrow{BC} = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$

$\overrightarrow{CA} = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{6}$

Let $x = \sqrt{35}$, $y = \sqrt{41}$ and $z = \sqrt{6}$

i.e., $x^2 = 35, y^2 = 41, z^2 = 6$

It is clear that, $x^2 + z^2 = 35 + 6 = 41 = y^2$

Pythagorous theorem is verified.

Therefore given vectors form right angled triangle.

2. Show that the points whose position vectors are $\vec{a} - 2\vec{b} + 3\vec{c}$, $2\vec{a} + 3\vec{b} - 4\vec{C}$ *and* $-7\vec{b} + 10\vec{c}$, *are collinear*.

Solution:

Let the points be A, B and C. and take O as origin.

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (\overrightarrow{2a} + \overrightarrow{3b} - \overrightarrow{4c}) - (\vec{a} - \overrightarrow{2b} + \overrightarrow{3c})$

$= \vec{a} + \overrightarrow{5b} - \overrightarrow{7c}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (-7\vec{b} + 10\vec{c}) - (2\vec{a} + 3\vec{b} - 4\vec{c})$

$= -2\vec{a} - 10\vec{b} + 14\vec{c} = -2(\vec{a} + 5\vec{b} - 7\vec{c})$

$= -2\overrightarrow{AB} = 2\overrightarrow{BA}$

That is, B is common in $\overrightarrow{BC}$ and $\overrightarrow{BA}$ and $\overrightarrow{BC}$ is parallel to $\overrightarrow{BA}$.

Therefore A,B and C are collinear.

4. Prove that the vector $\vec{a} = 4\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$, $\vec{b} = -2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $\vec{c} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ can form a triangle.

Solution:

$\vec{a} + \vec{b} = (4\hat{\imath} - 2\hat{\jmath} - 6\hat{k}) + (-2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}) = 2\hat{\imath} + \hat{\jmath} - \hat{k} = \vec{c}$

$\vec{a} + \vec{b} = \vec{c}$

Therefore, the vectors form a triangle.

5. Find unit vector parallel to the resultant of the vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

Solution:

Let $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Resultant of $\vec{a}$ and $\overrightarrow{b}$ $= \vec{a} + \vec{b} = \vec{R}$

$= 3\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$

$\therefore$ R = $|\vec{R}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = 7$

Therefore unit vector parallel to $\vec{R}$ is

$|\vec{R}| = \frac{\vec{R}}{R} = \frac{3\hat{\imath} + 6\hat{\jmath} - 2\hat{k}}{7}$

5.The position vector of four points P, Q, R and S are $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $3\hat{\imath} + \hat{\jmath}, 2\hat{\jmath} + 4\hat{\jmath} - 2\hat{k}$ and -$\hat{\imath} - 2\hat{\jmath} + \hat{k}$. show that $\boldsymbol{\overrightarrow{PQ}\ and\ \overrightarrow{RS}\ are\ parallel}$.

Solution:

$\overrightarrow{OP}$ = $2\hat{\imath} - \hat{\jmath} + \hat{k},$ $\quad \overrightarrow{OQ} = 3\hat{\imath} + \hat{\jmath}$

$\overrightarrow{OR} = 2\hat{\imath} + 4\hat{\jmath} - 2\hat{k},$ $\quad \overrightarrow{OS} = -\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ = $(3\hat{\imath} + \hat{\jmath}) - (2\hat{\imath} - \hat{\jmath} + \hat{k} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$ = $(-\hat{\imath} - 2\hat{\jmath} + \hat{k}) - (2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\overrightarrow{RS}$ = $-3(\hat{\imath} + 2\hat{\jmath} - \hat{k}) = -3\overrightarrow{PQ}$

Therefore, vectors are parallel.

6. Show that the vectors $9\hat{\imath} + \hat{\jmath} - 6\hat{k}$ and $4\hat{\imath} - 6\hat{\jmath} + 5\hat{k}$ $are\ orthogonal$.

Solution: Let $\vec{a} = 9\hat{\imath} + \hat{\jmath} - 6\hat{k},$ $\vec{b}$ = $4\hat{\imath} - 6\hat{\jmath} + 5\widehat{k}$

$\vec{a}.\vec{b} = 9 \times 4 + 1(-6) + (-6)(5) = 36 - 6 - 30;$ $\vec{a}.\vec{b}$ = 0

Therefore, vectors are orthogonal.

7. Find $\boldsymbol{\Lambda}$, if the vectors $3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$, and $6\hat{\imath} + \boldsymbol{\Lambda}\hat{\jmath} + 2\widehat{k}$ are orthogonal.

Solution: Let $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k},$ $\quad \vec{b} = 6\hat{\imath} + \hat{\jmath} + 2\hat{k},$

Given $\vec{a}, \vec{b}$ are perpendicular $\therefore \vec{a}.\overrightarrow{b}$ = 0

3(6) + 2($\boldsymbol{\Lambda}$) + (-5)(2) = 0

18+ 2$\boldsymbol{\Lambda}$ – 10 = 0

$\boldsymbol{2\,\Lambda}$ = - 8

$\therefore \lambda$ = - 4

Example8:

Find the cosine of the angle between vectors $3\hat{\imath} - \hat{\jmath} + \hat{k}$, and $\hat{\imath} + \hat{\jmath} - \hat{k}$.

Solution:

$|\vec{a}|$ = $\sqrt{3^2 + (-1)^2 + 1^2}$ = $\sqrt{11}$

$|\vec{b}|$ = $\sqrt{1^2 + 1^2 + (-1)^2}$ = $\sqrt{3}$

$\vec{a}.\vec{b}$ = (3)(1) + (-1)(1) + (1)(-1) = 3-1-1 = 1

Therefore cosine of the angle between $\vec{a}$ *and* $\vec{b}$ is given by,

Cos θ = $\frac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{b}|}$ (from dot product)

$\therefore$ cosθ = $\frac{1}{\sqrt{11}\,\sqrt{3}} = \frac{1}{\sqrt{33}}$

9. Find the projection of $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 3\hat{k}$ on $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$,

Solution:

Projection of $\vec{a}$ *on* $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{3(1)+1(-2)+3(1)}{\sqrt{1+4+1}} = \frac{4}{\sqrt{6}} = \frac{2\sqrt{6}}{3}$

Example10:

If $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, prove that $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$.

Solution:

$\vec{a} + \vec{b} = 3\hat{\imath} + 3\hat{k},$

$\vec{a} - \vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k},$

$((\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ = $(3\hat{\imath} + 3\hat{k})\ ((-\hat{\imath} + 2\hat{\jmath} + \hat{k},)$

= 3(-1) + 0 + 3 (1) = 0

Therefore $(\vec{a}+\vec{b})$ is perpendicular to $(\vec{a}-\vec{b})$

Example11:

If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$, find (i) $\vec{a} \times \vec{b}$, (ii) $2\vec{a} \times 3\vec{b}$ and (iii) $|a \times b|$

Solution:

i) $$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 2 & -1 & -1 \end{vmatrix} = \hat{\imath}(2+2) - \hat{\jmath}(-1-4) + \hat{k}(-1+2)$$

$$= 4\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$$

ii) $$2\vec{a} \times 3\vec{b} = \begin{vmatrix} i & j & k \\ 3 & -4 & 4 \\ 6 & -3 & -3 \end{vmatrix} = \hat{\imath}(12+12) - \hat{\jmath}(-9-24) + \hat{k}(-9+24)$$

$$= 24\hat{\imath} + 33\hat{\jmath} + 15\hat{k}$$

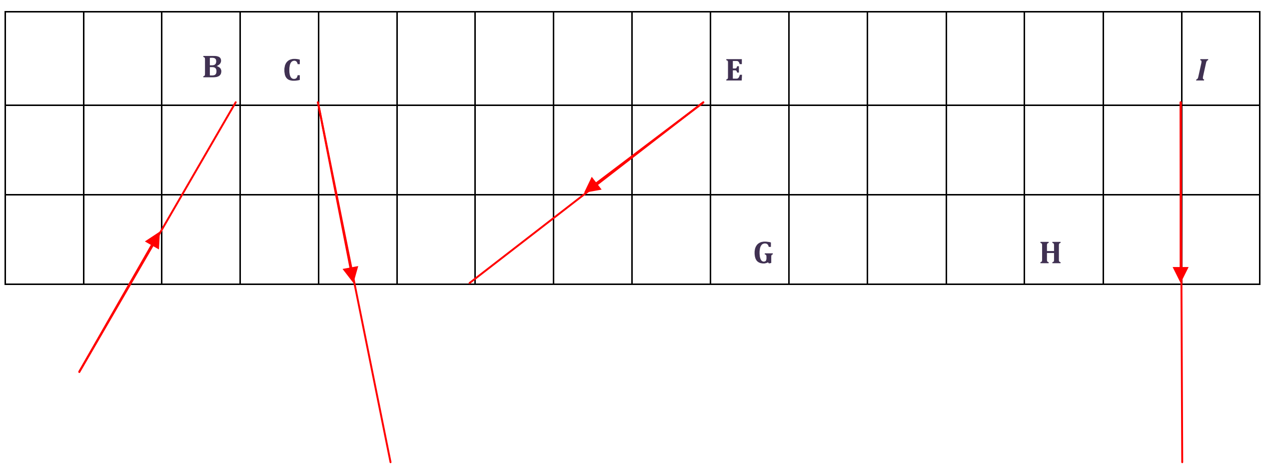
iii) $$\vec{a} \times \vec{b} = 4\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$$

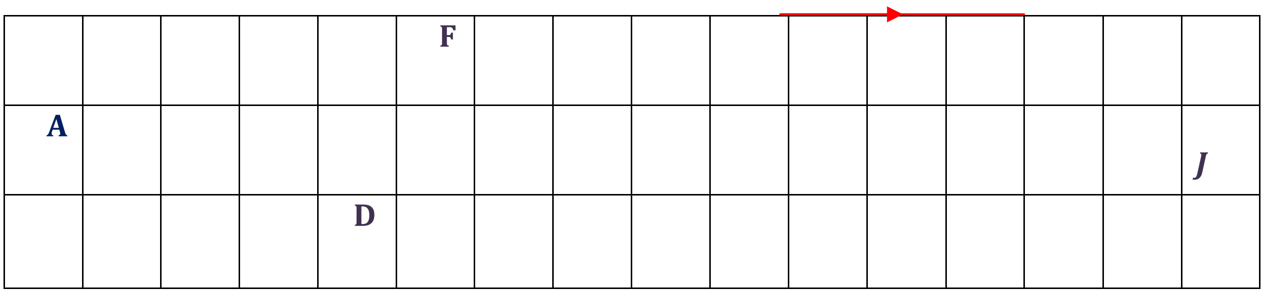
$$\therefore |\vec{a} \times \vec{b}| = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{50}$$

1.18 Column Vectors:

A Column vector is written in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where *x* is the horizontal component and *y* is the vertical component.

e.g.-





$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$, $\overrightarrow{EF} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$, $\overrightarrow{GH} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $\overrightarrow{IJ} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

1.19 LOWS OF COLUMN VECTORS:

For any two column vectors $\boldsymbol{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} r \\ s \end{pmatrix}$,

1. If $\boldsymbol{a} = \boldsymbol{b}$, then $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}$ and $p = r$ and $q = s$.

2. $\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p+r \\ q+s \end{pmatrix}$

   $\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p-r \\ q-s \end{pmatrix}$

3. $m\boldsymbol{a} = m\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} mp \\ mq \end{pmatrix}$ where m is a scalar .

4. $m\boldsymbol{a} + n\boldsymbol{b} = m\begin{pmatrix} p \\ q \end{pmatrix} + n\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} mp + nr \\ mq + ns \end{pmatrix}$ where m and n are scalars .

EXAMPLE12:

$p = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, $q = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$, $r = \begin{pmatrix} a \\ 10 \end{pmatrix}$, $s = \begin{pmatrix} 8 \\ b \end{pmatrix}$,

Find:

a) (i) $2p + 3q$ (ii) $p - 3q$

b) The value of 'a' if $2p = q + r$

c) The value of 'b' if q is parallel to s

d) $p - r + \frac{1}{2}s$.

SOLUTION:

a)(i) $2p + 3q = 2\begin{pmatrix} 6 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 12 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ -18 \end{pmatrix}$

$= \begin{pmatrix} 24 \\ -14 \end{pmatrix}$

(ii) $p - 3q = \begin{pmatrix} 6 \\ 2 \end{pmatrix} - 3\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 12 \\ -18 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 20 \end{pmatrix}$

b) $2p = q + r$

$2\begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} a \\ 10 \end{pmatrix}$

$\begin{pmatrix} 12 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 + a \\ 4 \end{pmatrix}$

Comparing the top component

$12 = 4 + a$

$\therefore a = 12 - 4$

$= 8$

c) Since q is parallel to s

$\mathbf{s} = k\mathbf{q}$

$\begin{pmatrix} 8 \\ b \end{pmatrix} = k\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$\begin{pmatrix} 8 \\ b \end{pmatrix} = \begin{pmatrix} 4k \\ -6k \end{pmatrix}$

$\therefore \; 8 = 4k$ ⟵ (Comparing the respective components)

$k = \frac{8}{4} = 2$

$b = -6k$

$= -6 \times 2$

$= -12$

d) $\mathbf{p} - \mathbf{r} + \frac{1}{2}\mathbf{s} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 8 \\ -12 \end{pmatrix}$ ⟵

Substitute $a = 8$ into $\mathbf{r} = \begin{pmatrix} a \\ 10 \end{pmatrix}$

and $b = -12$ into $\mathbf{s} = \begin{pmatrix} 8 \\ b \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 6 - 8 + 4 \\ 2 - 10 - 6 \end{pmatrix}$

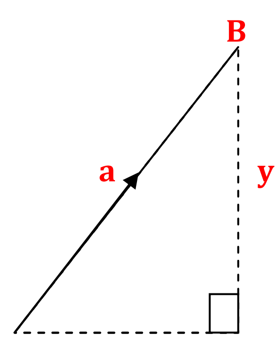
$= \begin{pmatrix} 2 \\ -14 \end{pmatrix}$

1.20 MAGNITUDE OF A COLUMN VECTOR:

The magnitude of a column vector $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ or $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ given by

$|\overrightarrow{AB}| = \sqrt{x^2 + y^2}$ Or

$|\mathbf{a}| = \sqrt{x^2 + y^2}$



A x C

EXAMPLE13:

$p = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $q = \begin{pmatrix} 7 \\ -6 \end{pmatrix}$ and $r = \begin{pmatrix} a \\ 9 \end{pmatrix}$.

Find:

a) $|P|$

b) 3p - q

c) |3p - q |

d) The possible values of a if $|q| = |r|$.

SOLUTION:

a) $|P| = \sqrt{3^2 + (-4)^2}$

$= \sqrt{25}$

= **5 units**

b) $3p - q = 3\begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 7 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ -12 \end{pmatrix} - \begin{pmatrix} 7 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -6 \end{pmatrix}$

c) $|3p - q| = \sqrt{2^2 + (-6)^2}$

$= \sqrt{40}$

= **6.32 units** (correct to 3 sig. fig.)

d) $|q| = |r|$

$\sqrt{7^2 + (-6)^2} = \sqrt{a^2 + 9^2}$

$$\sqrt{85} = \sqrt{a^2 + 81}$$

$85 = a^2 + 81$ ← Square both sides.

$a^2 = 4$

$a = \pm\sqrt{4}$ ← Take square root on both sides.

1.2 POSITIVE VECTORS:

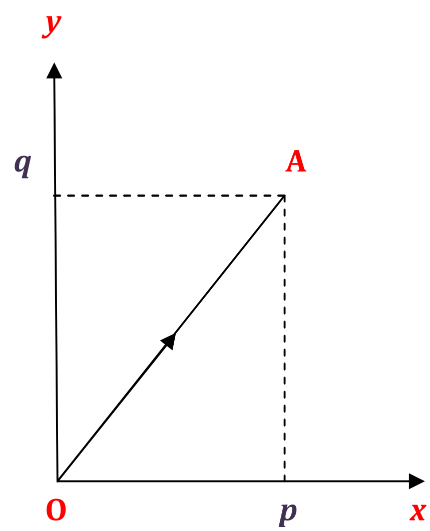
1. The position vector of any point *A* is the vector from the origin *O*, to that given point *A* .

   e.g. $\overrightarrow{OA}$ is the position vector *A* relative to *O* and is denoted by **a** .

   $\overrightarrow{OB}$ is the position vector *B* relative to *O* and is denoted by **a** .

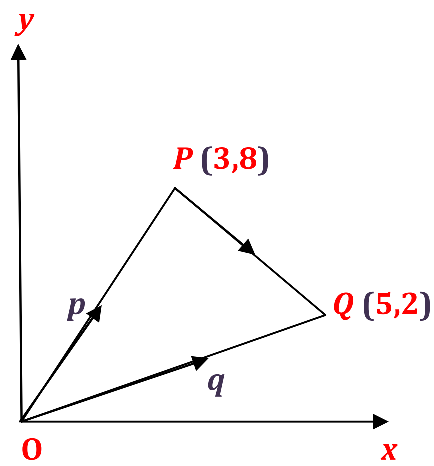
2. For any point *A* (p , q), the position vector of *A* with reference to the origin *O*, is given by :

$$\overrightarrow{OA} = \begin{pmatrix} p \\ q \end{pmatrix}$$



3. For any two point A and B, $\overrightarrow{AB}$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$



e.g.-

The vector $\overrightarrow{PQ}$ can be expressed as

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= q - p$

The position vectors of the coordinates P (3, 8) and Q(5, 2) are $\overrightarrow{OP} = \binom{3}{8}$ and $\overrightarrow{OQ} = \binom{5}{2}$.

EXAMPLE14:

Given that A is the point (2, 1), B is the point (7, 3) and $\overrightarrow{BC} = \binom{-4}{1}$.

a) Express $\overrightarrow{AB}$ as a column vector.

b) Find the coordinates of the point C.

c) Find the coordinates of the point D if $ABDC$ is a parallelogram.

SOLUTION:

a) $\overrightarrow{OA} = \binom{2}{1}$, $\overrightarrow{OB} = \binom{7}{3}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \binom{7}{3} - \binom{2}{1}$

$= \binom{5}{2}$

b) $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\overrightarrow{OC} = \overrightarrow{BC} - \overrightarrow{OB}$

$= \binom{-4}{1} + \binom{7}{3}$

$= \binom{3}{4}$

$\therefore$ $C = (3, 4)$

c) Let the coordinates of D be (h, k).

$\overrightarrow{OD} = \binom{h}{k}$

$\overrightarrow{OD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= \binom{h}{k} - \binom{3}{4}$

$= \binom{h}{k}$

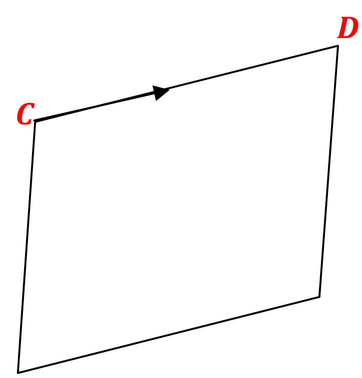
Since $ABCD$ is a parallelogram,

$\overrightarrow{CD} = \overrightarrow{AB}$

$\binom{h-3}{k-4} = \binom{5}{2}$

$\therefore$ $h - 3 = 5$

$h = 8$

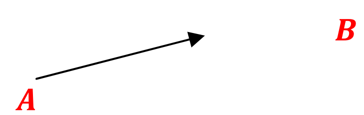


$k - 4 = 2$

$k = 6$

$\overrightarrow{OD} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

$\therefore \quad D = (8, 6)$



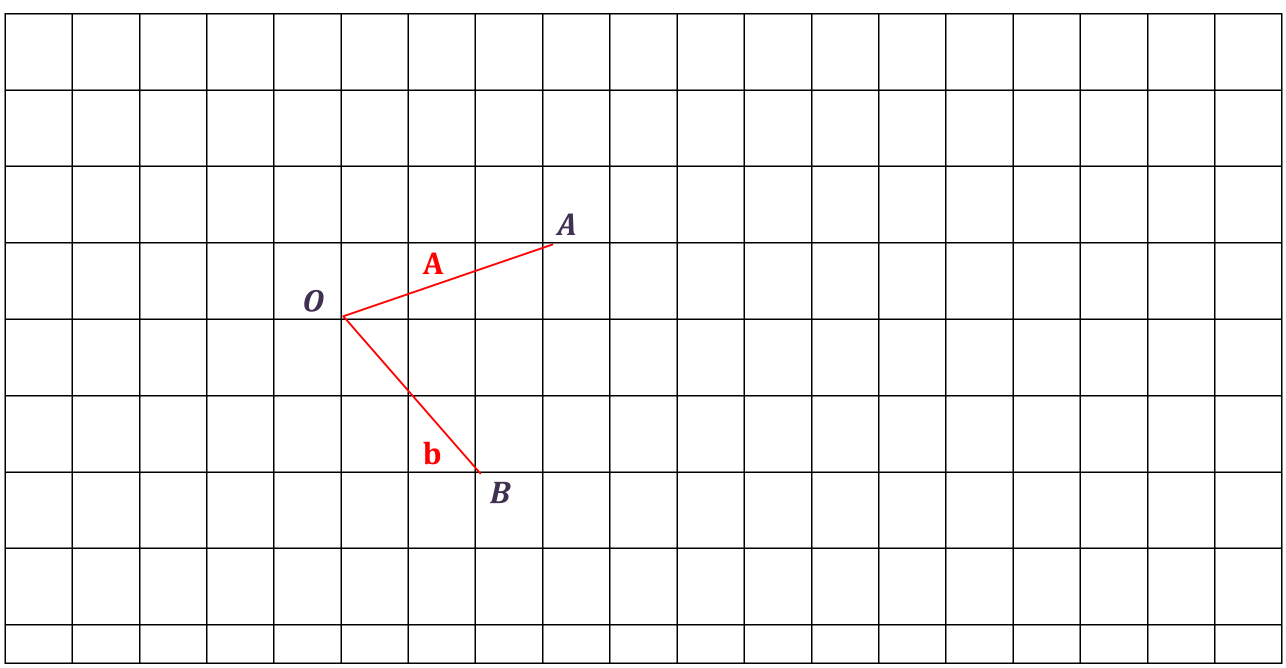
EXAMPLE15:

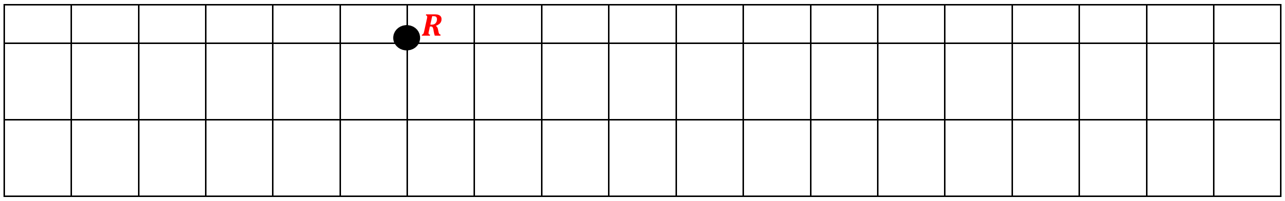
In the diagram, $\overrightarrow{OA} = a$ **and** $\overrightarrow{OB} = b$

a) Mark and label clearly on the grid, the point P **such that** $\overrightarrow{OP} = 2a$ **and the point** Q **such that** $\overrightarrow{OQ} = 2a + 3b$ **.**

b) What is the special name given to the quadrilateral $OBQP$ **?**

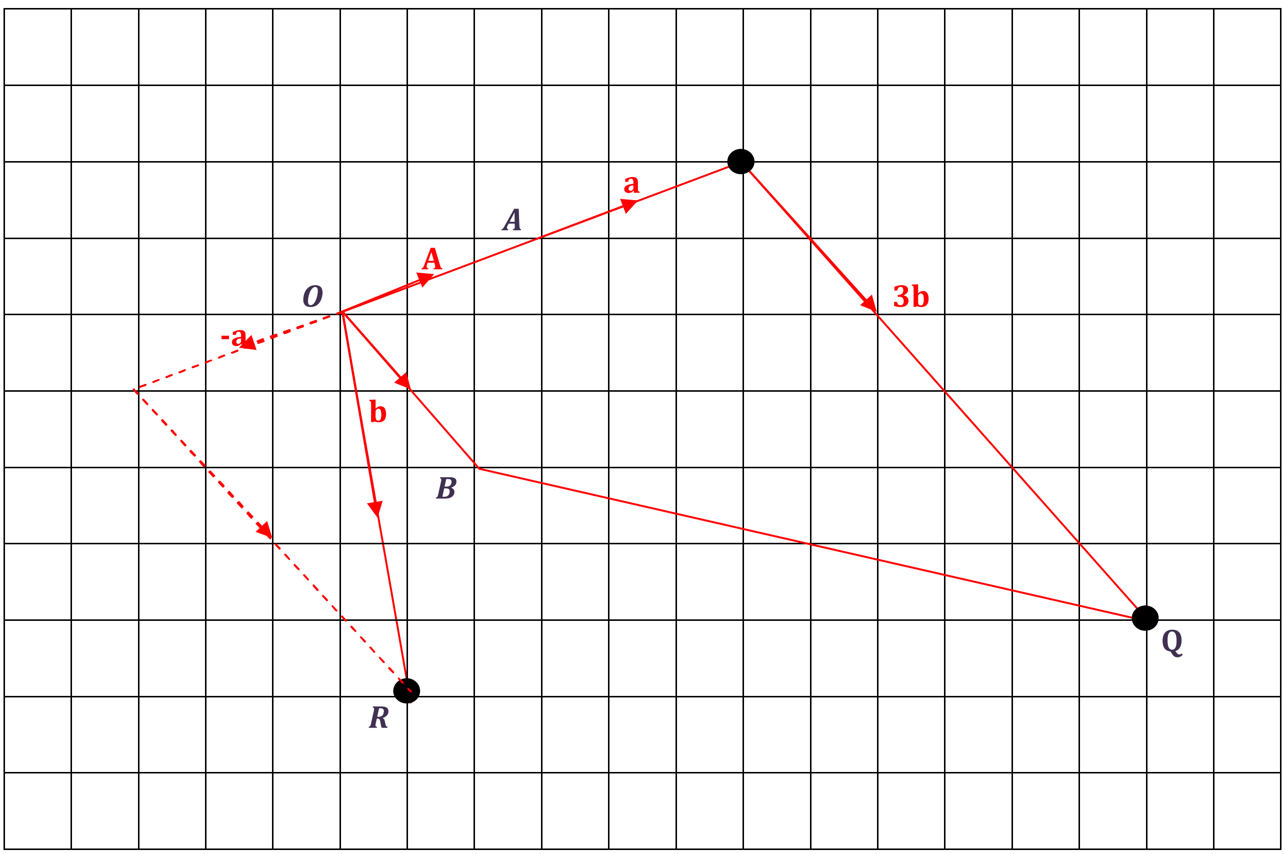
c) Write down $\overrightarrow{OR}$ **is term of** a **and** b





SOLUTION:

a)



b) ***OBQP*** **is a trapezium** ←

A trapezium is a quadrilateral with a pair of parallel opposite

c) $\overrightarrow{OR} = a + 2b$

EXAMPLE 16:

ABCD is a parallelogram. Given that $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AD}$, $\overrightarrow{AF} = \frac{1}{4}\overrightarrow{AC}$, $\overrightarrow{AB}$ = p and $\overrightarrow{AD}$ = q and *F* lies on the intersection of *AC* and *BE* .

a) Find in terms of p and q,

i) $\overrightarrow{AC}$

ii) $\overrightarrow{AF}$

iii) $\overrightarrow{BE}$

iv) $\overrightarrow{BF}$

b) Show that $\overrightarrow{BF} = \frac{3}{4}\overrightarrow{BE}$.

c) Find the numerical value of

i) $\frac{FE}{BF}$,

ii) $\frac{Area\ of\ triangle\ AFE}{Area\ of\ triangle\ ABF}$,

iii) $\frac{Area\ of\ triangle\ BFC}{Area\ of\ triangle\ BAC}$,

iv) $\frac{Area\ of\ triangle\ BFC}{Area\ of\ triangle\ ABCD}$.

SOLUTION:

a) i) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$ (Parallelogram law)

$= p + q$

ii) $\overrightarrow{AF} = \frac{1}{4}\overrightarrow{AC}$

$= \frac{1}{4}(p + q)$

$= \frac{1}{4}p + \frac{1}{4}q$

iii) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$ (Triangle law)

$= \overrightarrow{AB} + \frac{1}{3}\overrightarrow{AD}$

$= p + \frac{1}{3}q$

iv) $\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$ (Triangle law)

$= p + \frac{1}{4}p + \frac{1}{4}q$

$= -\frac{3}{4}p + \frac{1}{4}q$

b) $\overrightarrow{BF} = -\frac{3}{4}p + \frac{1}{4}q$.

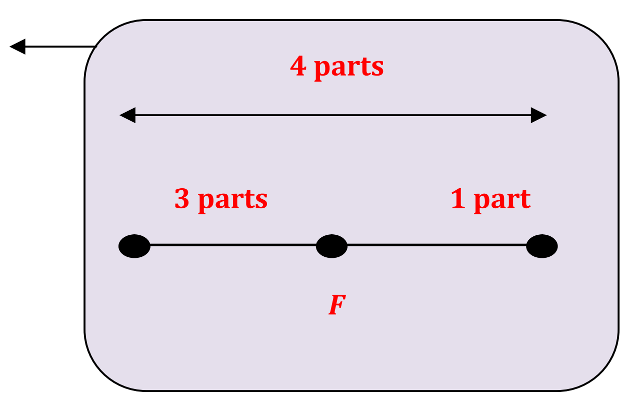
$= \frac{3}{4}(-p + \frac{1}{3}q)$

$= \frac{3}{4}\overrightarrow{BE}$

$\therefore \overrightarrow{BF} = \frac{3}{4}\overrightarrow{BE}$

c) i) $\overrightarrow{BF} = \frac{3}{4}\overrightarrow{BE}$

$\therefore \frac{FE}{BF} = \frac{1}{3}$



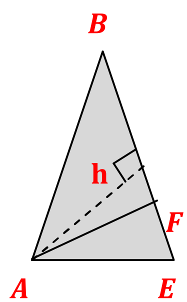
ii) $\frac{Area\ of\ \Delta AFE}{Area\ of\ \Delta ABF}$

$$= \frac{\frac{1}{2} X FE X h}{\frac{1}{2} X BF X h}$$

$$= \frac{FE}{BF}$$

$$= \frac{1}{3}$$

TIP FOR STUDENTS:



$\Delta\ AFE$ and $\Delta\ ABF$ share a common height, h units

iii) $\frac{Area\ of\ \Delta BFC}{Area\ of\ \Delta BAC}$

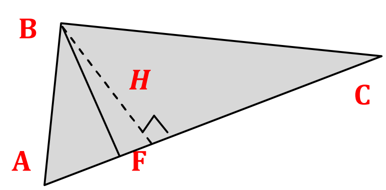
$$= \frac{\frac{1}{2} X FC X H}{\frac{1}{2} X AC X H}$$

$$= \frac{FC}{AC}$$

$$= \frac{3}{4}$$

TIP FOR STUDENTS:

$\Delta\ BFC$ and $\Delta\ BAC$ share a common height, H units



iv) $\frac{Area\ of\ \Delta\ BFC}{Area\ of\ \Delta\ ABCD}$

$= \frac{Area\ of\ \Delta\ BFC}{2\ X\ Area\ of\ \Delta\ BAC}$

$= \frac{3}{2\ X\ 4}$

$= \frac{3}{8}$

17.) If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \widehat{2\imath} - \widehat{2\jmath} + \hat{k}$, find unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Also find sine of the angle between the vectors.

SOLUTION:

Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ = $\frac{\vec{a}\ X\ \vec{b}}{|\vec{a}\ X\ \vec{b}|}$

$$\vec{a}\ X\ \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \hat{\imath}\,(3) + \hat{\jmath}\,(-1) + \hat{k}\,(-4)$$

$$= 3\,\hat{\imath} + \hat{\jmath} + 4\,\hat{k}$$

$$|\vec{a}\ X\ \vec{b}| = \sqrt{9+1+16} = \sqrt{26}$$

Therefore unit vector perpendicular to both

$\vec{a}$ and $\vec{b}$ = $\frac{\vec{a}\ X\ \vec{b}}{|\vec{a}\ X\ \vec{b}|}$ = $\frac{3\,\hat{\imath} + \hat{\jmath} + 4\,\hat{k}}{\sqrt{26}}$

Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then,

$$\sin\theta = \frac{|\vec{a}\ X\ \vec{b}|}{|\vec{a}|\ X\ |\vec{b}|}$$ **(From vector product)**

$$|\vec{a}\ X\ \vec{b}| = \sqrt{26}$$

$|\vec{a}| = \sqrt{1+1+1} = \sqrt{3}$, $|\vec{b}| = \sqrt{4+4+1} = \sqrt{3}$

$\therefore \sin\theta = \frac{\sqrt{26}}{\sqrt{3.3}} = \sqrt{\frac{26}{27}}$

18.) Find the area of the triangle, whose two sides are $\vec{a} = \widehat{2\imath} - \widehat{3\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - \widehat{2k}$.

SOLUTION:

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \hat{\imath}(6-1) - \hat{\jmath}(-4-1) + \hat{k}(2+3)$$

$$= 5\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{25+25+25} = \sqrt{75} = 5\sqrt{3}$$

$$\text{Area of Triangle} = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{5\sqrt{3}}{2} \text{ sq . units .}$$

19.) Find the area of the parallelogram, whose adjacent sides are $\widehat{2\imath} + \widehat{2\jmath} + \widehat{3k}$ and are $\widehat{3\imath} + \hat{\jmath} - \widehat{2k}$.

SOLUTION :

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 2 & 3 \\ 3 & 1 & -2 \end{vmatrix} = i(-4-3) - j(-4-9) + k(2-6)$$

$$= -7\,i + 13\,j + 4\,k$$

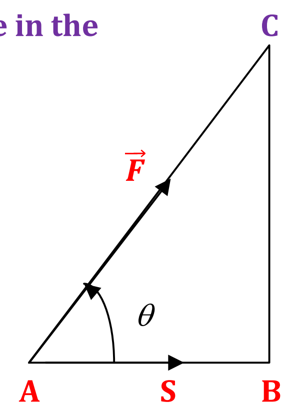
$|\vec{a} \times \vec{b}| = \sqrt{49+169+16} = \sqrt{234}$ **sq. units.**

Area of parallelogram

$= \frac{1}{2} |\vec{a} \times \vec{b}| = \sqrt{234}$ **sq. units.**

1.22 WORK DONE:

Work done by a force is defined as the product of the force in the direction of displacement and displacement .



Suppose a force $\vec{F}$ displaces an object from A to B, through a displacement $\vec{S}$, then

Work Done = **Magnitude of force $\vec{F}$ in the direction of $\vec{S}$ X magnitude of $\vec{S}$**

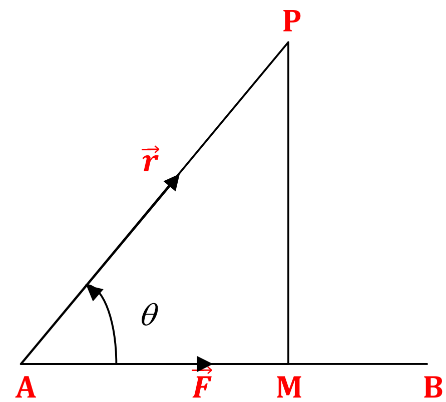
$= |\vec{F}| .\cos |\vec{S}| = |\vec{F}| \ |\vec{S}| . \cos \theta$

Work Done = $\vec{F} . \vec{S}$

1.23 MOMENT OF A FORCE ABOUT A POINT:

Moment of a force $\vec{F}$ about a point P is defined as " *The product of Magnitude of force and perpendicular distance of $\vec{F}$ from P* ".

Let $\vec{F}$ be the force. Then to find moment of $\vec{F}$ about P.



i.e., **Magnitude** of Moment of a force $\vec{F}$ about P

$= |\vec{F}| . PM = |\vec{F}| \, |\vec{PA}| . \sin\theta$

$= |\vec{F}| . |\vec{r}| . \sin\theta$

$= |\vec{r} \times \vec{F}|$

Moment of $\vec{F}$ about P is a vector $\vec{r} \times \vec{F}$ or $|\vec{r} \times \vec{F}|$ accordingly as rotation is in anticlockwise or clockwise

$\therefore$ Moment of $\vec{F}$ about P $= \pm \vec{r} \times \vec{F}|$

Magnitude of $\vec{F}$ about P $= |\vec{r} \times \vec{F}|$

20.) Find the work done by the force $\vec{F} = 2i + j - 2k$ in the direction of $i + j + k$.

SOLUTION:

Let $\vec{S} = i + j + k$

Work Done $= \vec{F} . \vec{S}$

$\therefore$ Work done $= (2i + j - 2k) . (i + j + k)$

$= 2(1) + (1)(1) - (-2)(1)$

$= 2 + 1 + -2$

$= 1$

21.) A force $\vec{F}$ = 2i + j - 2k acting on particle at (3, 2, 2) displaces it to the point (1, 3, -1), find the work done .

SOLUTION:

Take A = (3, 2, 2) , B = (1, 3, -1)

i.e., $\overrightarrow{OA}$ = 3i + 2j + 2k , $\overrightarrow{OB}$ = i + 3j - k

Now $\vec{S}$ = $\overrightarrow{AB}$ = $\overrightarrow{OB}$ - $\overrightarrow{OA}$

= (i + 3j – k) - (3j + 2j + 2k)

= (2i + j – 3k)

Work Done = $\vec{F}$. $\vec{S}$

∴ Work done = (2i + j - 2k) . (- 2i + j + 3k)

= 2 (-2) + (1) (1) + (- 2) (- 3)

= - 4 + 1 + -6

= 3

22.) A force $\vec{F}$ = 2i + j - 2k acting on point (- 3, 2, 1) find the magnitude of the moment of force $\vec{F}$ about the point (2, 1, 2) .

SOLUTION:

Take $A = (-3, 2, 1)$, $B = (2, 1, 2)$

$\therefore$ $\overrightarrow{OA} = 3i + 2j + 2k$, $\overrightarrow{OP} = 2i + j - 2k$

$\therefore$ $\vec{r} = \overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP}$

$= (-3i + 2j + k) - (2i + j + 2k)$

$\vec{r} = -5i + j - k$

Moment of Force $\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -5 & 1 & -1 \\ 2 & +1 & 1 \end{vmatrix} = i(2) - j(-3) + k(-1)$

$= -2i + 3j + 3k$

$\therefore$ **Magnitude of moment of force**

$|\vec{M}| = |\vec{r} \times \vec{F}| = \sqrt{2^2 + 3^2 + (7)^2} = \sqrt{62}$

23.) A force $\vec{F} = 3i + 2j + k$ acts at a point a whose position vector is $2i - j$. Find the moment of $\vec{F}$ about the origin .

SOLUTION:

Take **O** as the origin, so that

$$= \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -5 & 1 & -1 \\ 2 & +1 & 1 \end{vmatrix} = - i - 2j - k$$

$$= -2i + 3j + 3k$$

$\therefore$ **Its magnitude** $= |\vec{r} \times \vec{F}| = \sqrt{1+4+1} = \sqrt{6}$

24.) Show that vectors 2i - j + k, i - 3j + 5k and 3i - 4j - 4k from sides of right angled triangle .

SOLUTION:

Let $\vec{AB}$ = 2i - j + k, $\vec{BC}$ = i - 3j + 5k

$\vec{AC}$ = 3i - 4j - 4k

$\vec{AB} + \vec{BC}$ = 2i - j + k + i - 3j + 5k

= 3i - 4j - 4k = $\vec{CA}$

This means, $\vec{AB}$, $\vec{BC}$ and $\vec{CA}$ from sides of a triangle . Now we have to prove the triangle ABC is a right angled triangle .

Magnitude of $\vec{AB} = |\vec{AB}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6} = a$

Magnitude of $\vec{BC} = |\vec{BC}| = \sqrt{(1)^2 + (-3)^2 + -5^2} = \sqrt{35} = b$

Magnitude of $\vec{CA}$ = $|\vec{CA}|$ = $\sqrt{(3)^2+(-4)^2+(-4)^2}$ = $\sqrt{41}$ = c

Now, $a^2 + b^2$ = 6 + 35 = 41

The triangle ABC is a right angled triangle.

25.) If $\vec{a}$ = 3i - j - 4k , $\vec{b}$ = - 2i + 4j - 3k and $\vec{c}$ = i + 2j - k find unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$.

SOLUTION:

$3\vec{a} - 2\vec{b} + 4\vec{c}$ = 9i - 3j - 12k + 4i - 8j + 6k + 4i + 8j - 4k

= 17i - 3j - 10k

Unit vector parallel to $3\vec{a} - 2\vec{b} + 4\vec{c}$

$$= \frac{3\vec{a} - 2\vec{b} + 4\vec{c}}{\sqrt{3\vec{a} - 2\vec{b} + 4\vec{c}}}$$

$$= \frac{17i - 3j - 10k}{\sqrt{(17)^2 + (-3)^2 + (-10)^2}}$$

$$= \frac{17i - 2j - 10k}{\sqrt{398}}$$

26.) Show that the three points whose position vector are 2i - 3j + 4k , i - 2j + 3k and 7j + 10k are collinear .

SOLUTION:

Let **O** be any reference point and **A**, **B**, **C** be given points. The position vectors are

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \quad \text{(By definition of p . v . of a point)}$$

$$= (i - 2j + 3k) - (2i + 3j - 4k)$$

$$= - i – 5j + 7k$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \overrightarrow{OC} - \overrightarrow{OA}$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$

$$= (- 7j + 10k) - (2i + 3j - 4k) = 2i - 10j + 14k$$

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = - i – 5j + 7k - i - 5j + 7k - (- 2i - 10j + 14k)$$

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

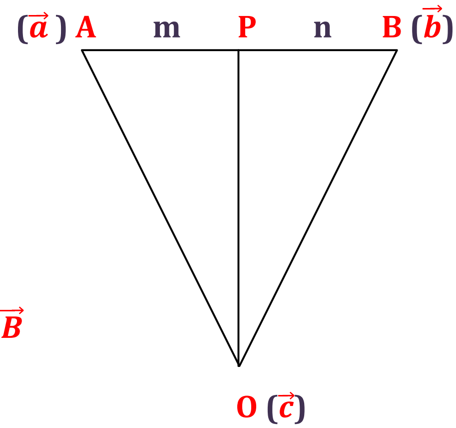
This means that the given vectors are collinear.

27.) Find the position vector of a point which divides the line joining the two points.

SOLUTION:

Let **A** and **B** be two points and **P** divides the line **AB** in the ratio **m : n** (**Fig.**). Let **O** be any reference point

Then $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$, $\overrightarrow{OP}$ = ?



From Fig, $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$.

BUT $\frac{\overrightarrow{AP}}{\overrightarrow{AB}} = \frac{m}{\overrightarrow{m+n}}$ $\overrightarrow{OP} = \overrightarrow{OA} + \frac{m}{\overrightarrow{m+n}}\overrightarrow{AB}$

$$= \vec{a} + \left(\frac{m}{\overrightarrow{m+n}}\right)(\vec{a} - \vec{b})$$

$$\overrightarrow{OP} = \frac{\vec{a}(m+n) + \mathrm{m}(\vec{b} - \vec{a})}{\overrightarrow{m+n}}$$

$$= \frac{\mathrm{m}\,\vec{b} + \mathrm{n}\vec{a})}{\overrightarrow{m+n}}$$

i.e.- P. V. of a point, which divides the line joining the two given points

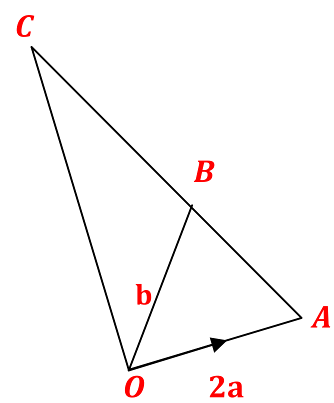
$$\frac{\mathrm{m}\,\vec{b} + \mathrm{n}\,\vec{a}}{\overrightarrow{m+n}}$$

If m = n, then P will be mid point of AB.

Then $\overrightarrow{OP} = \frac{\vec{a} + \vec{b}}{2}$

EXAMPLE28:

a) In the diagram, $\overrightarrow{OA} = 2a$, $\overrightarrow{OB} = b$.



b) B and C are points such that $\overrightarrow{BC} = 2a - 4a$

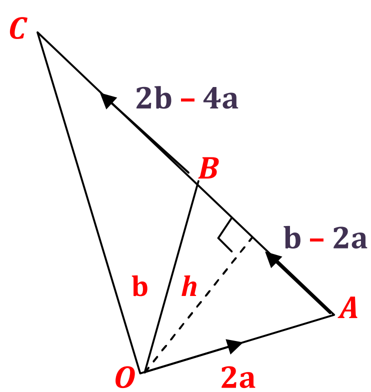
i) Find $\overrightarrow{OC}$ in terms of a and b

ii) Explain why *A* , *B* and *C* lie in a straight line .

iii) Find the ratio $|\overrightarrow{AB}| : |\overrightarrow{AC}|$.

iv) Calculate the area of triangle *OAB* given that the area of triangle *OAC* is 27 units^2.

SOLUTION:



a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= b - 2a$

b) i) $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$= b + 2b - 4a$

$= 3b - 4a$

ii) $\overrightarrow{BC} = 2b - 4a$

To show that $\overrightarrow{BC} \; // \; \overrightarrow{AB}$, prove that

$\overrightarrow{BC} = k\,\overrightarrow{AB}$ Where k is a real number

$= 2(b - 2a)$

$= 2\overrightarrow{AB}$

$\therefore \overrightarrow{BC} \; // \; \overrightarrow{AB}$

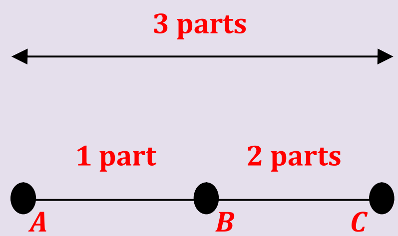
iii) $|\overrightarrow{AB}| : |\overrightarrow{AC}|$

$= |\overrightarrow{AB}| : 3|\overrightarrow{AB}|$

$= 1 : 3$

From (b) (ii), $BC = 2AB$.

$\therefore AC = 3AB$



iv) Δ OAB and Δ OAC share a co

$$\frac{\text{Area of } \Delta\, OAB}{\text{Area of } \Delta\, OAC} = \frac{\frac{1}{2} X \not{A}BX\not{h}}{\frac{1}{2} X \not{A}CX\not{h}}$$

$$= \frac{AB}{AC} = \frac{1}{3}$$

$$\frac{\text{Area of } \Delta\, OAC}{27} = \frac{1}{3}$$

Area of OAC $= \frac{1}{3} \text{ X } 27$ ← Area of OAC = 27 units^2 (Given)

$= 9$ units^2

EXAMPLE29:

The diagram shows trapezium $OACB$. $\overrightarrow{OA}$ = a and $\overrightarrow{OB}$ = b. $\overrightarrow{AC} = 3\,\overrightarrow{OB}$, $\overrightarrow{BE} = \frac{2}{3}\,\overrightarrow{BC}$

and OE meets AB at D.

a) Express as simple as possible in terms of a and b.

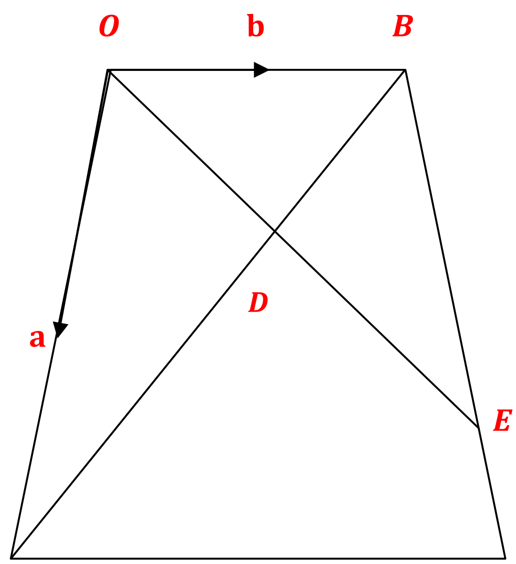
i) $\overrightarrow{BA}$,   ii) $\overrightarrow{BE}$,   iii) $\overrightarrow{OE}$.

b) Given that $\overrightarrow{BD} = \frac{2}{9}\ \overrightarrow{BA}$, show that $\overrightarrow{OD}\ = \frac{2}{9}$ a + $\frac{7}{9}$ b.

c) Find the numerical value of

i) $\frac{Area\ of\ triangle\ BDE}{Area\ of\ triangle\ BOD}$.

ii) $\frac{Area\ of\ triangle\ BDE}{Area\ of\ triangle\ AOD}$.



SOLUTION:

a)

i) $\overrightarrow{BA}\ = \overrightarrow{OA}\ - \overrightarrow{OB}$

$= \text{a} - \text{b}$

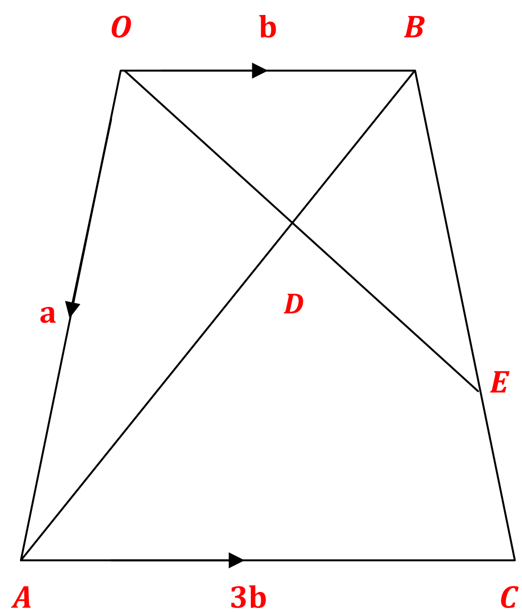
ii) $\overrightarrow{BA}\ = \overrightarrow{OA}\ - \overrightarrow{OB}$

$= \text{a} - \text{b} + 3\text{b}$

$= \text{a} + 2\text{b}$

$\overrightarrow{AC} = \overrightarrow{OB}$

$= 3\text{b}$



$\overrightarrow{BE} = \frac{2}{3}\overrightarrow{BC}$

$= \frac{2}{3}(a + 2b)$

$= \frac{2}{3}a + \frac{4}{3}b$

iii) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= b + \frac{2}{3}a + \frac{4}{3}b$

$= \frac{2}{3}a + \frac{7}{3}b$

b) i) $\overrightarrow{BD} = \frac{2}{9}\overrightarrow{BA}$

$= \frac{2}{9}(a - b)$

$= \frac{2}{9}a - \frac{2}{9}b$

$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= b + \frac{2}{9}a - \frac{4}{9}b$

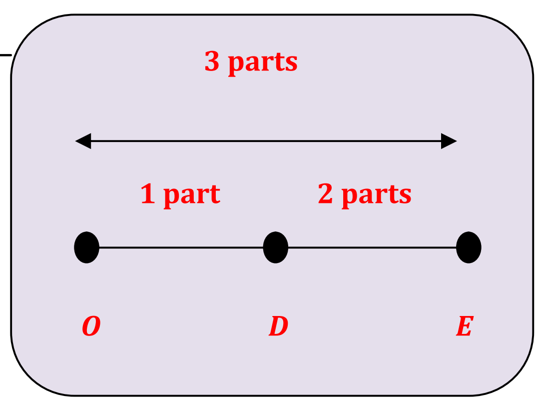
$= \frac{2}{9}a + \frac{7}{9}b$

c) i) $\overrightarrow{OE} = \frac{2}{3}a + \frac{7}{3}b$

$= 3(\frac{2}{9}a + \frac{7}{9}b)$

$= \overrightarrow{OD}$

$\therefore OE = 3OD$

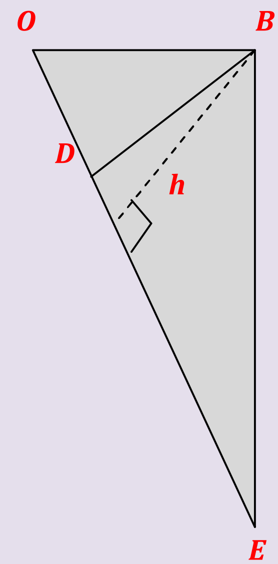


$$\frac{Area\ of\ \Delta\ BDE}{Area\ of\ \Delta\ BOD} = \frac{\frac{1}{2} X AB X h}{\frac{1}{2} X AC X h}$$

$$= \frac{DE}{OD}$$

$$= \frac{2}{1}$$

$$= 2$$



Δ *BDE* and Δ *BOD* share a common height, *h* units .

ii) $\frac{Area\ of\ \Delta\ BDE}{Area\ of\ \Delta\ AOD}$

$$= \frac{Area\ of\ \Delta\ BDE}{Area\ of\ \Delta\ BOD} \times \frac{Area\ of\ \Delta\ BOD}{Area\ of\ \Delta\ AOD}$$

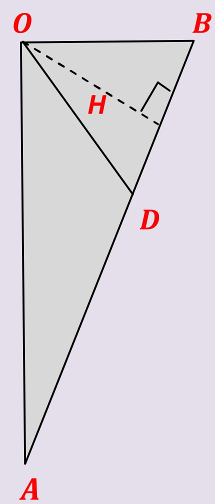
$$= 2 \times \frac{BD}{AD}$$

$$= 2 \times \frac{2}{7}$$

$$= \frac{4}{7}$$

$BD = \frac{2}{9} BA$

$\therefore \frac{BD}{AD} = \frac{2}{7}$



Δ BOD and ΔAOD share a common height, *H* units

$$\frac{Area\ of\ \Delta\ BOD}{Area\ of\ \Delta\ AOD} = \frac{\frac{1}{2} X BD X H}{\frac{1}{2} X AD X H}$$

$$= \frac{BD}{AD}$$

EXAMPLE30:

In the diagram, ***OPRQ*** is a parallelogram and ***PQT*** is a straight line . ***OQ*** is produced to

meet **RT** at **S** .

a) Given that $\overrightarrow{OP}$ = p , $\overrightarrow{OQ}$ = q and $\overrightarrow{QT}$ = 3 (q – p) ,express, as simply as possible, in

terms of **p** and **q** ,

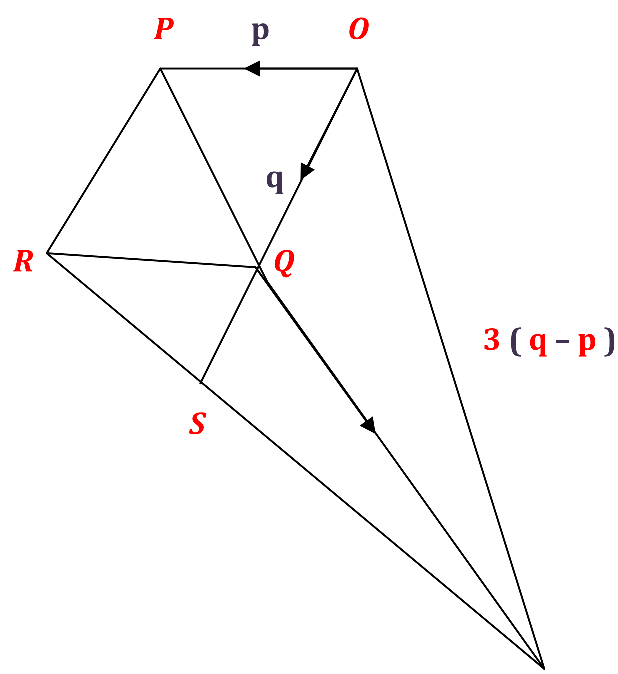
i) $\overrightarrow{OP}$ ii) $\overrightarrow{OP}$ iii) $\overrightarrow{OP}$ iv) $\overrightarrow{OP}$

b) Show that triangle ***TQS*** is similar to triangle ***TPR***.

c) Find the numerical value of

i) $\dfrac{\textit{Area of } \Delta\, TQS}{\textit{Area of } \Delta\ \text{TPR}}$

ii) $\dfrac{\textit{Area of } \Delta\, OPQ}{\textit{Area of } \Delta\ \text{OQT}}$



T

SOLUTION:

a) i) $\overrightarrow{QT} = \overrightarrow{OQ} + \overrightarrow{QT}$

$= q + 3(q - p)$

$= q + 3q - 3p$

$= 4q - 3p$

ii) $\overrightarrow{PT} = \overrightarrow{OT} - \overrightarrow{OP}$

$= 4q - 3p - p$

$= 4q - 4q$

iii) $\overrightarrow{OR} = \overrightarrow{OP} - \overrightarrow{OR}$ (Parallelogram law)

$= p + q$

iv) $\overrightarrow{RT} = \overrightarrow{OT} - \overrightarrow{OR}$

$= 4q - 3p - (p + q)$

$= 4q - 3p - p - q$

$= 3q - 4p$

b) $\angle TQS = \angle TPR$ (corr. $\angle$s $QS // PR$)

$\angle TSQ = \angle TRP$ (corr. $\angle$s $QS // PR$)

$\therefore$ Δ TQS is similar to Δ TPR . (Shown)

Two triangles are similar if two of their corresponding angles are equal.

c) i) $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{QP}$

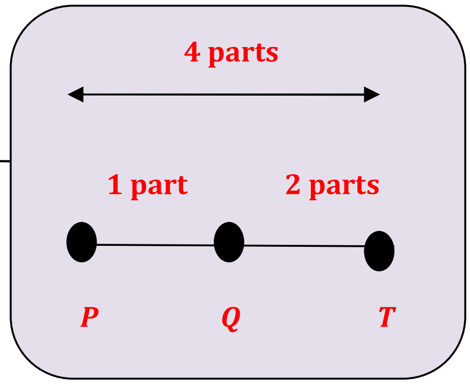
$= \mathbf{q} - \mathbf{p}$

$\overrightarrow{QT} = 3(\mathbf{q} - \mathbf{p})$

$= 3\,PQ$

$\therefore 3\,PQ = QT$

$\frac{PQ}{QT} = \frac{1}{3}$



$\therefore \frac{PQ}{QT} = \frac{3}{4}$

$$\frac{\text{Area of } \Delta\, TQS}{\text{Area of } \Delta\ TPR} = \frac{QT}{PT}\left(\frac{QT}{PT}\right)^2$$

Using similar triangles,

$$\left(\frac{A_1}{A_2}\right) = \left(\frac{l_1}{l_2}\right)^2$$

$$= \left(\frac{3}{4}\right)^2$$

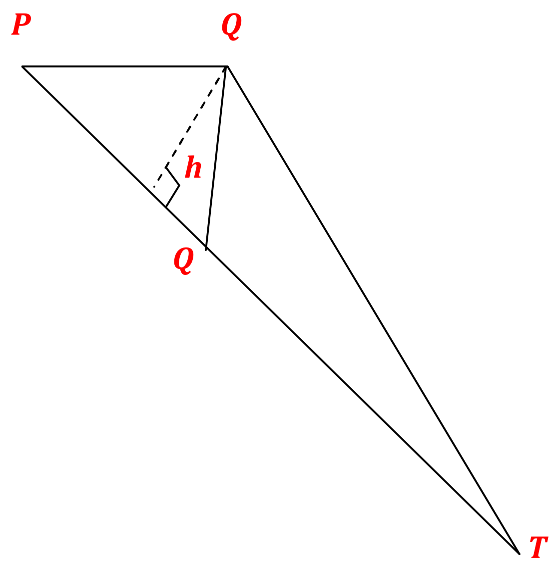
$$= \frac{9}{16}$$

ii) $\Delta\, OPQ$ and $\Delta\, OQT$ share a common height, h units .

$$\frac{\textit{Area of } \Delta\, OPQ}{\textit{Area of } \Delta\ OQT} = \frac{\frac{1}{2} XPQXh}{\frac{1}{2} XQTXh}$$

$$= \frac{PQ}{QT}$$

$$= \frac{1}{3}$$



SUMMARY AND KEY POINTS

1. In applied mathematics, there exists two types of physical quantities called *scalars* and *vectors*. Those quantities which involve only magnitudes and no direction are called *scalars*. Those quantities which involve both magnitude and direction are called *vectors*.

Distance, Area, Volume, Mass, Speed, Temperature are *scalars* and Velocity, Acceleration, Force, Electric field etc are *vectors*.

2. REPRESENTATION OF A VECTOR:

Vector quantity required length. In a precise manner, vector means **"** A directed line segment **"** . It is represented by a line OP directed from an initial point O to the terminal point P and denoted by $\overrightarrow{OP}$. Hence the length of the vector $\overrightarrow{OP}$ denoted by

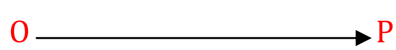


Figure (1)

$|\overrightarrow{OP}|$ **=** OP is called magnitude or modulus of the vector . The arrow mark in Figure (1)

$\overrightarrow{OP}$ denoted direction if a = op, it represents length then $\vec{a} = \overrightarrow{op}$, represents vector. Arrow mark on the head of a quantity represents a vector .

3. KINDS OF VECTORS:

a. NULL VECTORS:

A vector having initial and terminal point co – incident is called as null (or zero) vector . In other wards a vector with zero magnitude is called Null Vector.

b. UNIT VECTOR:

A vector having its modulus as unity is called unit vector . If $\vec{a}$ is any vector and $|\vec{a}| = 1$, then $\vec{a}$ is called Unit Vector is denoted by

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

c. EQUAL VECTORS:

Two vector having same magnitude and same direction are called Equal Vectors .

Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal if

$$|\vec{a}| = |\vec{b}|$$

d. EQUAL VECTORS:

If a is any given vector, then - $\vec{a}$ is called Negative of $\vec{a} \cdot \vec{a}$ and - $\vec{a}$ and have same magnitude but opposite direction .

e. SCALAR MULTIPLE OF A VECTORS:

If $\vec{a}$ is any vector and λ is any scalar, then $\lambda \vec{a}$ called *Scalar multiple* of $\vec{a}$.

Scalar multiple of a vector $\vec{a}$ is a vector whose magnitude is scalar times the magnitude of $\vec{a}$.

If $|\vec{a}| = 2$, $|2\vec{a}| = 2 \text{ X } 2 = 4$, $|\frac{1}{4}\vec{a}| = \frac{1}{4} \text{ X } 2 = \frac{1}{2}$

f. COLLINEAR VECTORS:

Two vectors parallel to the same line regardless of their magnitude and direction are called collinear. If $\vec{a}$ and $\vec{b}$ are any two collinear vectors, there exist λ (a scalar) such that $\vec{b} = \lambda \vec{a}$.

i.e., one of the two collinear vectors can be expressed as scalar multiple of the other .

g. COPLANAR VECTORS:

A system of vectors lying in the parallel planes or lie in the same plane are said to be Coplanar Vector .

Evidently any two vectors always are Coplanar.

g. LIKE AND UNLIKE VECTORS:

Two vectors, which are collinear and have same direction are called Like Vecrors.

The vectors whose direction are opposite are called Unlike Vectors.

4. POSITION VECTOR OF A POINT (P.V. OF A POINT):

If A,B,C are any three points in a plane and O is fixed point in the plane. Then $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ are called position vectors of points A,B,C respectively with reference to O.The position vector of a point specifies the position of a point relative to an arbitrary chosen point .

5. ADDITION OF VECTORS:

Addition of vectors is given by well known law of parallelogram of forces. It states that

" If two vectors acting at a point are represented by the adjacent sides of a parallelogram, then their resultant is given by the diagonal of the parallelogram passing through that point ".

Two forces acting at a point O can be represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ as adjacent sides and diagonal $\overrightarrow{OC}$ represents resultant as diagonal of the parallelogram OACB .

i.e., $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$ Or $\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$

The same geometrical construction is used in defining vector addition.

If Δ ABC is any triangle then,

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

Figure (2)), similarly, $\overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$

In addition of vectors, initial point of the second vector coincides with terminal point of the first vector. Then sum is obtained by joining initial point of the first to terminal point of the second vector.

6. SUBTRACTION OF VECTORS:

Since $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, $\overrightarrow{AB} - \overrightarrow{AC} = -\overrightarrow{BC}$

$$\therefore \quad \overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$$

In subtraction of vectors, the initial point of two vectors coincide.

$\overrightarrow{OP} - \overrightarrow{OQ} = \overrightarrow{QP}$, $\overrightarrow{OM} - \overrightarrow{OL} = \overrightarrow{LM}$

Addition of vectors: $\vec{a}$, $\vec{b}$, $\vec{c}$, satisfies the following basic laws.

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
3. $(m + n)\vec{a} = m\vec{a} + n\vec{a}$
4. $m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$
5. $m(n\vec{a}) = mn\vec{a}$
6. $\vec{a} + \vec{o} = \vec{o} + \vec{a} = \vec{a}$

7. COMPONENTS OF A VECTOR IN 2 MUTUALLY PERPENDICULAR DIRECTIONS (i.e.,IN PLANE):

It is convention that $\vec{i}$, $\vec{j}$ are taken as unit vectors in the positive directions of x and y axis. Let P(x, y) be any point. Then, $\overrightarrow{OP}$ is called the position vector of the point P.

Draw PM ⊥ x - axis. Join OP. Now OM = x and unit vector in the direction of OM is $\hat{\imath}$.

$\therefore \overrightarrow{OM} = x\hat{\imath}$

MP = y and unit vector in the direction of y is $\hat{\jmath}$, $\therefore \overrightarrow{MP} = y\hat{\jmath}$

MP = y and unit vector in the direction of y is $\hat{\jmath}$, $\therefore \overrightarrow{MP} = y\hat{\jmath}$

Now, $\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = x\hat{\imath} + y\hat{\jmath}$

i.e., the position vector of the point p(x, y) is

$$\vec{r} = \overrightarrow{OP} = x\hat{\imath} + y\hat{\jmath}$$

The length of $\overrightarrow{OP} = |\vec{r}| = \sqrt{OM^2 + MP^2}$

$$\therefore |x\hat{\imath} + y\hat{\jmath}| = \sqrt{x^2 + y^2}$$

The direction of $\overrightarrow{OP}$ with x – axis is given by

$$tan\ \theta\ =\ \frac{y}{x} \quad \text{or} \quad \theta = \tan^{-1}\frac{y}{x}$$

8. PRODUCT OF TWO VECTORS:

There are two kinds of products, one is called scalar or dot product

and other vector or cross product.the scalar or dot product of two vectors a and b is denoted by $\vec{a}.\vec{b}$ or $\vec{a}\,\vec{b}$ and their vector or cross product is $\vec{a} \times \vec{b}$ or $[\vec{a}\,\vec{b}]$

(a)Dot product: Let $\vec{a}$ = (a_1, a_2, a_3), $\vec{b}$ = (b_1, b_2, b_3) be two vectors. then,dot product of $\vec{a}$ and $\vec{b}$ is defined by $a_1b_1 + a_2b_2 + a_3b_3$.

$$\therefore \quad \vec{a}.\overrightarrow{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\vec{a}.\overrightarrow{b} = \vec{b}.\overrightarrow{a}$$

b.) Vector product:

Let $\vec{a}$ =(a_1,a_2,a_3), $\vec{b}$ = (b_1,b_2,b_3)

Then, the cross product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \times \vec{b} = \hat{\imath}\,(a_2b_3 - a_3b_2) - \hat{\jmath}\,(a_1b_3 - a_3b_1) + \hat{k}\,(a_1b_2 - a_2b_1)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

b_1

Vector product is not commutative.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \end{vmatrix} = - \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ b_1 & b_2 & b_3 \end{vmatrix} = - \vec{b} \times \vec{a}$$

$b_1 \quad b_2 \quad b_3 \qquad\qquad a_1 \quad a_2 \quad a_3$

9.) GEOMETRICAL MEANING OF SCALAR AND VECTOR PRODUCTS:

Let $\vec{a}$ and $\vec{b}$ be any two non-zero vectors.The dot product of $\vec{a}$ and $\vec{b}$ is equal to product of magnitude of $\vec{a}$ and $\vec{b}$ and cosine of the angle between them.

i.e., $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|.\cos\theta$

Also $\vec{a}.\vec{b} = |\vec{a}|$ x projection of $\vec{b}$ in the direction of $\vec{a}$.

a.) $\theta = 90^0$, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|.\cos 90^0$

$$\vec{a}.\vec{b} = 0$$

i.e.,vectors are perpendiculars to each other. Perpendicular vectors are also called orthogonal vectors.

b.) If $\vec{a}$ is parallel to $\vec{b}$ (or coincident with $\vec{b}$), then $\theta = 0$

$\therefore\ \vec{a}.\vec{b} = |\vec{a}||\vec{b}| \qquad\qquad \vec{a}.\vec{a} = |\vec{a}|^2$

c.)For unit vectors, $\hat{\imath}, \hat{\jmath}\ and\ \hat{k},\ \ \hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$. Since $\hat{\imath}, \hat{\jmath}, \hat{k}$ are mutually perpendicular to each other.

$\hat{\imath}.\hat{\imath} = |\hat{\imath}|^2 = 1$ similarly, $\hat{\jmath}.\hat{\jmath} = 1$, $\hat{k}.\hat{k} = 1$

10.) GEOMETRICAL MEANING OF $\vec{a}$ x $\vec{b}$.

The vector product of two non-zero vector $\vec{a}$ and $\vec{b}$ is given by $\vec{a}$ x $\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where θ is angle between them and $\hat{n}$ is a unit vector in the direction of $\vec{a}$ x $\vec{b}$.

i.e. $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$

Where $\vec{a}$, $\vec{b}$ and $\vec{a} \times \vec{b}$ form a right handed system i.e.$\hat{n}$ is the unit vector, perpendicular to both $\vec{a}$ and $\vec{b}$.

$\therefore \vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta|\hat{n}| = |\vec{a}||\vec{b}| \sin\theta$ $\quad (|\hat{n}| = 1)$

= OA.OB.sinθ

= OA x perpendicular from B to A

= Area of parallelogram OACB

Thus, $\vec{a} \times \vec{b}$ is a vector, whose modulus gives the area of parallelogram OACB.

IMPORTANT FOR STUDENTS:

g) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

h) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

i) $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}| = |\vec{a}||\vec{b}|.\sin\theta$

j) If θ= 0, sin θ = 0, $\therefore \vec{a} \times \vec{b} = 0$, i.e. $\therefore \vec{a} \times \vec{a} = 0$ i.e. vector are co-incident.

k) If $\vec{a}$ is perpendicular to $\vec{b}$, then $\theta = 90^0$, $\therefore$ $\sin\theta = \sin 90^0 = 1$

$$\therefore |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$$

l) For unit vector : i,j,k, $\quad$ i×i = j×j = k×k = 0 and ixj=k, jxk=i, kxi=j

11.AREA OF A TRIANGLE GIVEN TWO OF ITS SIDES $\vec{a}$ AND $\vec{b}$:

Let $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AC} = \vec{b}$ and $\lfloor BAC = \theta$ (radians)

Area of ABC = ½ x AB x CD

= ½ XAB.AC sinθ

$= \frac{1}{2} |\overrightarrow{AB}||\overrightarrow{AC}| \sin\theta$

$= \frac{1}{2} |\vec{a}||\vec{b}|.\sin\theta$

Area of Δ ABC = $\frac{1}{2}|\vec{a} \times \vec{b}|$, where $\vec{a}$ and $\vec{b}$ are two sides.

12. AREA OF PARALLELOGRAM:

Area of parallelogram = AB X DE

$$= |\vec{a}||\vec{b}|.\sin\theta = |\vec{a} \times \vec{b}|$$

Thus, $|\vec{a} \times \vec{b}|$ represents area of parallelogram with $\vec{a}$ and $\vec{b}$ as adjacent sides.

13. PROJECTION OFONE VECTOR ON ANOTHER:

$\overrightarrow{LM}$ and $\overrightarrow{CD}$ are two vectors. Let θ be the angle between them.

Here, CA and DB ⊥ LM. Then AB is called the projection of $\overrightarrow{CD}$ on $\overrightarrow{LM}$.

$Take\ \overrightarrow{LM} = \vec{a} \quad and \quad \overrightarrow{CD} = \vec{b}$

Projection of $\overrightarrow{CD}$ on $\overrightarrow{LM}$

= AB = CE = CD$\cos\theta$

$= |\overrightarrow{CD}|.\cos\theta = \frac{|\vec{b}|.|\vec{a}|.cos\theta}{|\vec{a}|}$

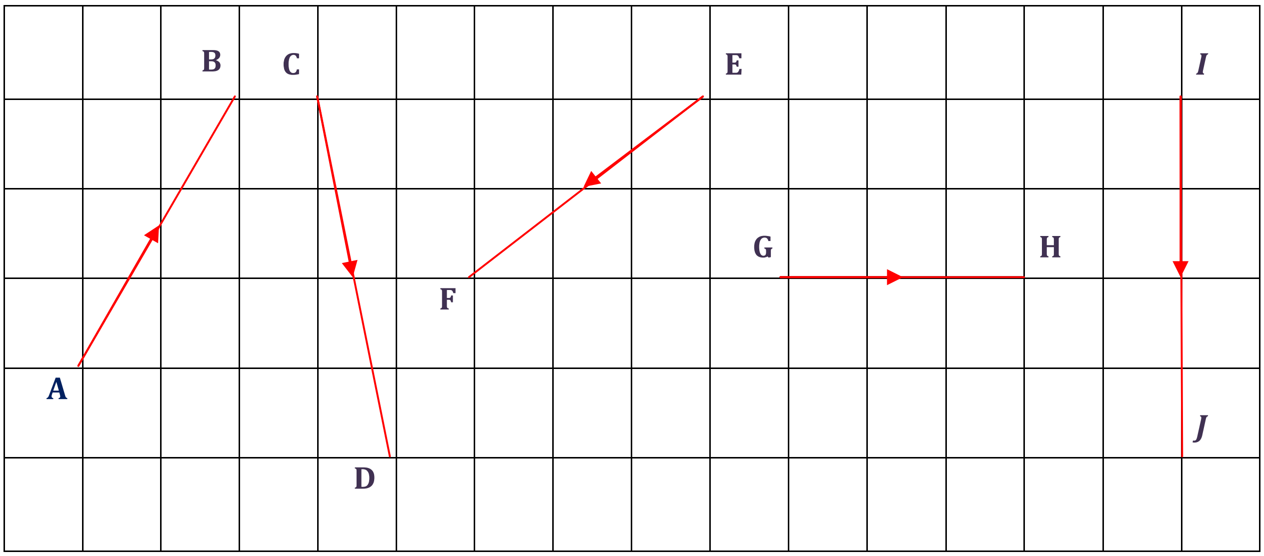
Projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

And projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

14. Column Vectors:

A Column vector is written in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where x is the horizontal component and y is the vertical component.

e.g.-



$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$, $\overrightarrow{EF} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$, $\overrightarrow{GH} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $\overrightarrow{IJ} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

15. LOWS OF COLUMN VECTORS:

For any two column vectors $a = \begin{pmatrix} p \\ q \end{pmatrix}$ and $b = \begin{pmatrix} r \\ s \end{pmatrix}$,

1. If $a = b$, then $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}$ and $p = r$ and $q = s$.
2. $a + b = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$

$\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p-r \\ q-s \end{pmatrix}$

3. $m\boldsymbol{a} = m\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} mp \\ mq \end{pmatrix}$ **where** m **is a scalar .**

4. $m\boldsymbol{a} + n\boldsymbol{b} = m\begin{pmatrix} p \\ q \end{pmatrix} + n\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} mp + nr \\ mq + ns \end{pmatrix}$ **where** m and n **are scalars .**

16. MAGNITUDE OF A COLUMN VECTOR:

The magnitude of a column vector $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ **or a** = $\begin{pmatrix} x \\ y \end{pmatrix}$ **given by**

$|\overrightarrow{AB}| = \sqrt{x^2 + y^2}$ **Or**

$|\boldsymbol{a}| = \sqrt{x^2 + y^2}$

17. POSITIVE VECTORS:

a) The position vector of any point A **is the vector from the origin** O**, to that given point** A **.**

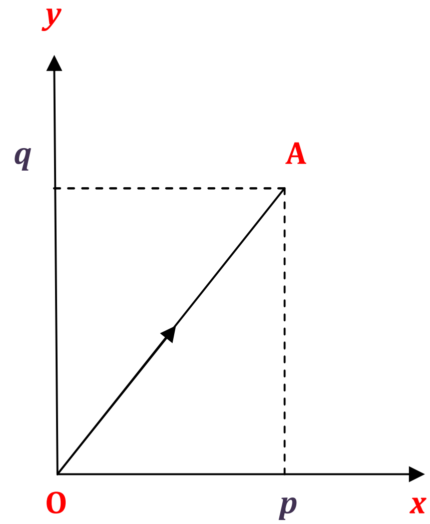
e.g. $\overrightarrow{OA}$ **is the position vector** A **relative to** O **and is denoted by a .**

$\overrightarrow{OB}$ **is the position vector** B **relative to** O **and is denoted by a .**

b) For any point A **(p , q), the position vector of** A **with reference to the origin** O**, is**

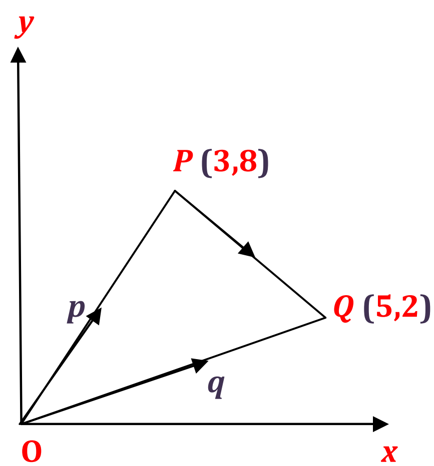
given by :

$\overrightarrow{OA} = \begin{pmatrix} p \\ q \end{pmatrix}$



c.) For any two point A and B, $\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$



e.g.-

The vector $\overrightarrow{PQ}$ can be expressed as

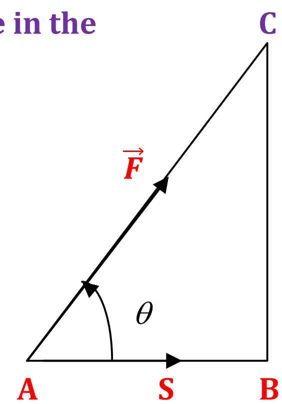
$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= \mathbf{q} - \mathbf{p}$

The position vectors of the coordinates ***P*** **(3, 8)** and ***Q*** **(5, 2)** are $\overrightarrow{OP} = \binom{3}{8}$ and $\overrightarrow{OQ} = \binom{5}{2}$.

18. WORK DONE:

Work done by a force is defined as the product of the force in the direction of displacement and displacement .



Suppose a force $\vec{F}$ displaces an object from A to B, through a displacement $\vec{S}$, then

Work Done = Magnitude of force $\vec{F}$ in the direction of $\vec{S}$ X magnitude of $\vec{S}$

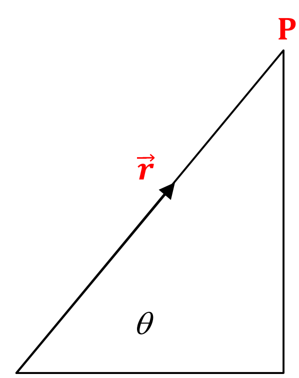
$= |\vec{F}| .\cos |\vec{S}| = |\vec{F}| \ |\vec{S}| . \cos \theta$

Work Done = $\vec{F} . \vec{S}$

19.) MOMENT OF A FORCE ABOUT A POINT:

Moment of a force $\vec{F}$ about a point P is defined as " *The product of Magnitude of force and perpendicular distance of $\vec{F}$ from P* ".

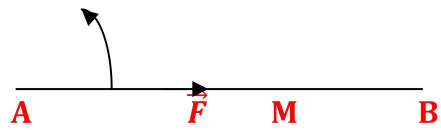
Let $\vec{F}$ be the force. Then to find moment of $\vec{F}$ about P.



i.e., Magnitude of Moment of a force $\vec{F}$ about P

$= |\vec{F}| . PM = |\vec{F}| \; |\overrightarrow{PA}| . \sin\theta$

$= |\vec{F}| . \; |\vec{r}| \; . \sin\theta$



$= |\vec{r} \text{ X } \vec{F}|$

Moment of $\vec{F}$ about P is a vector $\vec{r}$ X $\vec{F}$ or $|\vec{r}$ X $\vec{F}|$ accordingly as rotation is in anticlockwise or clockwise

∴ Moment of $\vec{F}$ about P $= \pm\vec{r} \text{ X } \vec{F}|$

Magnitude of $\vec{F}$ about P $= |\vec{r} \text{ X } \vec{F}|$