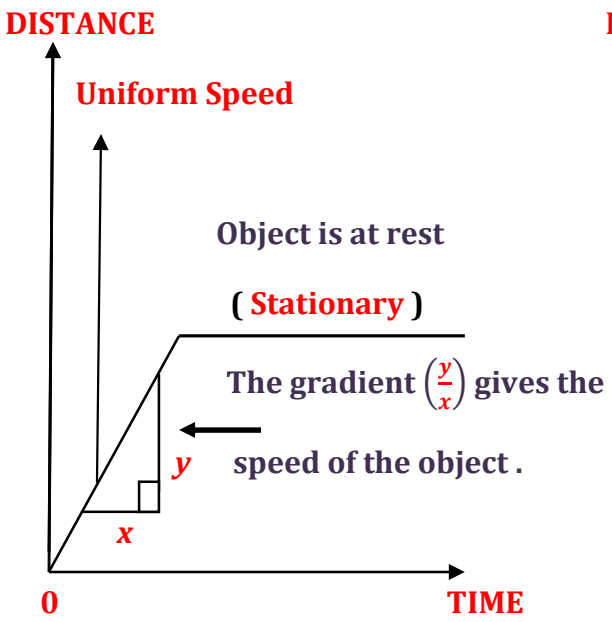
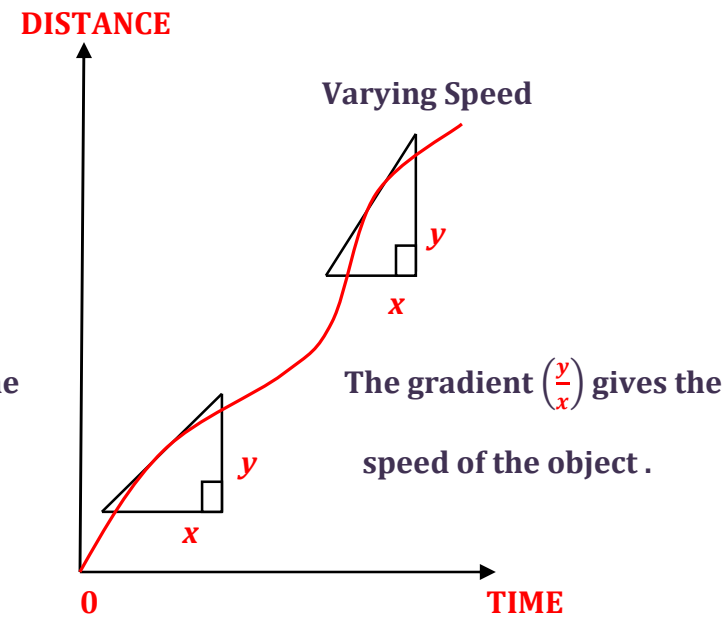
KINEMATICS / TRAVEL GRAPHS / CONVERSION GRAPHS

1.1 KINEMATICS / TRAVEL GRAPHS:

DISTANCE – TIME GRAPHS:

The gradient of a distance – time graph gives the instantaneous speed of a moving object .





$$\text{AVERAGE SPEED} = \frac{\textit{Total Distance Travelled}}{\textit{Total Time Taken}}$$

EXAMPLE1:

Andrew drove from home to the post office to send a parcel. On his way back, he stopped at the library to return a book . The travel graph shows his journey.

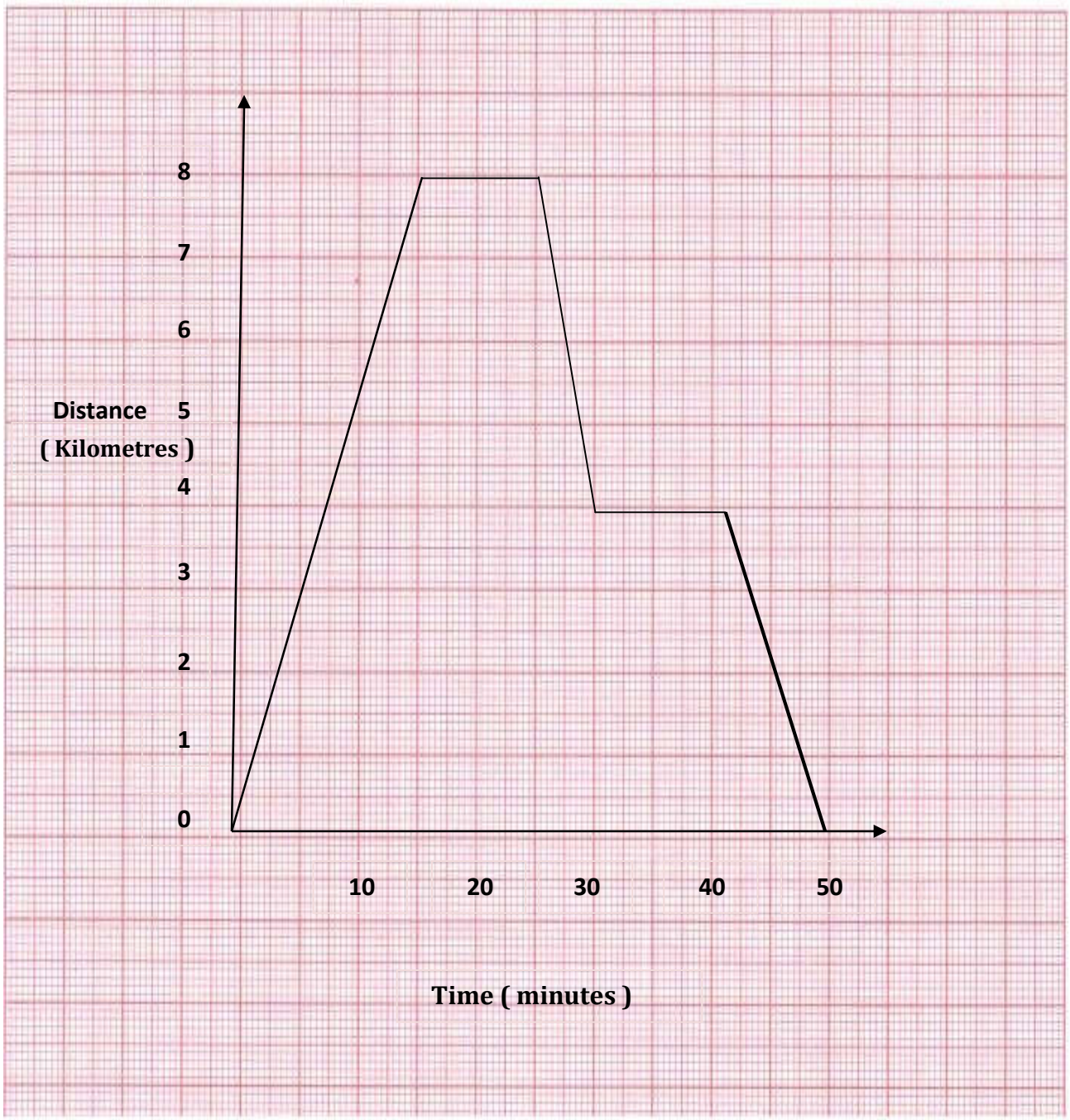
a) What was his speed in the first 15 minutes ? Gives your answer in kilometers per hour.

b) How far is the post office from his home?

c) How long was he at the library?

d) What was his speed when t = 27?

e) What was his average speed for the whole journey?


8
7
6
Distance 5
(Kilometres)
4
3
2
1
0
10
20
30
40
50
Time (minutes)

SOLUTION:

a) Speed (First 15 minutes)

The gradient of the first part of the graph gives the speed at the time .

$= \frac{8\ km}{\frac{1}{4}h}$

$= 32$ km / h

Total time taken

$= 15$ min

$= \frac{15}{60}$ h

$= \frac{1}{4}$ h

b) The post office is 8 km from his home.

c) $42 - 30 = 12$ min

He was at the library for 12 minutes.

d) Speed (when $t = 27$)

The gradient of the graph from $t = 25$ to $t = 30$ gives the speed when $t = 27$.

$= \frac{4\ km}{\frac{1}{12}h}$

$= 348$ km / h

Total time taken

$= 5$ min

$= \frac{5}{60}$ h

$= \frac{1}{12}$ h

e) Average speed from the whole journey

$= \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$= \frac{16\ km}{\frac{5}{6}h}$

$= 19.2$ km / h

Total distance travelled

$= 8 + 8$

$= 16$ km

Total time taken

$= 50$ min

$= \frac{50}{60}$ h

$= \frac{5}{6}$ h

EXAMPLE2:

The travel graph show the journey of two trains *P* and *Q* travelling between towns *A*, *B* and *C* . Town *A* and Town *B* are 100 km apart while Town *B* and Town *C* are 150 km apart . Train *P* leaves Town *A* at 08 00 and arrives at Town *C* at 12 00, Train *Q* leaves Town *A* at 09 00 and arrives at Town *A* at 12 00 .

FIND

a) The speed of train *P* from

i) 08 00 to 09 30 ,

ii) 09 30 to 10 00 ,

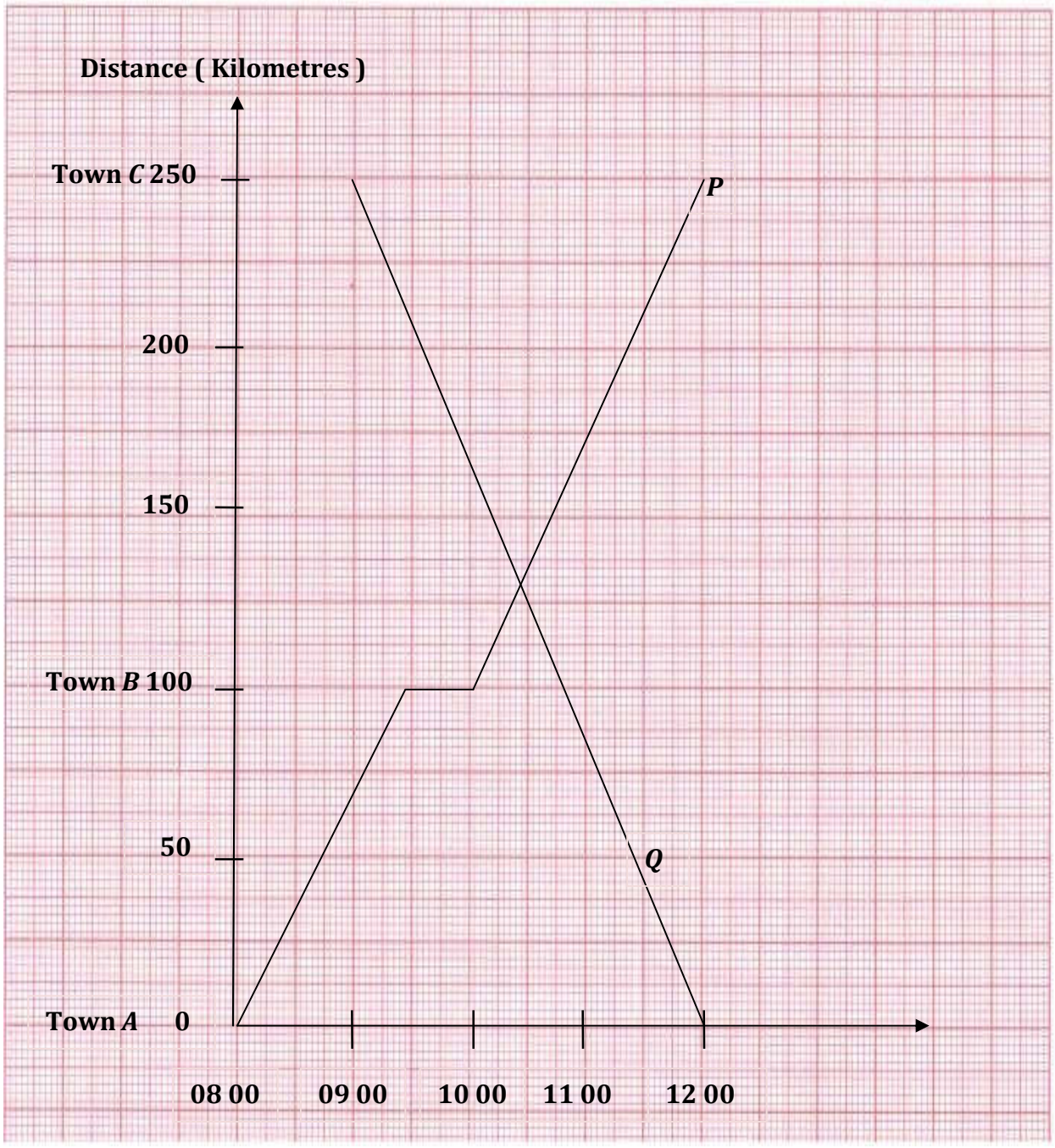
iii) **10 00** **to** **12 00** **.**

b) **The average speed of Train *P* for the whole journey ,**

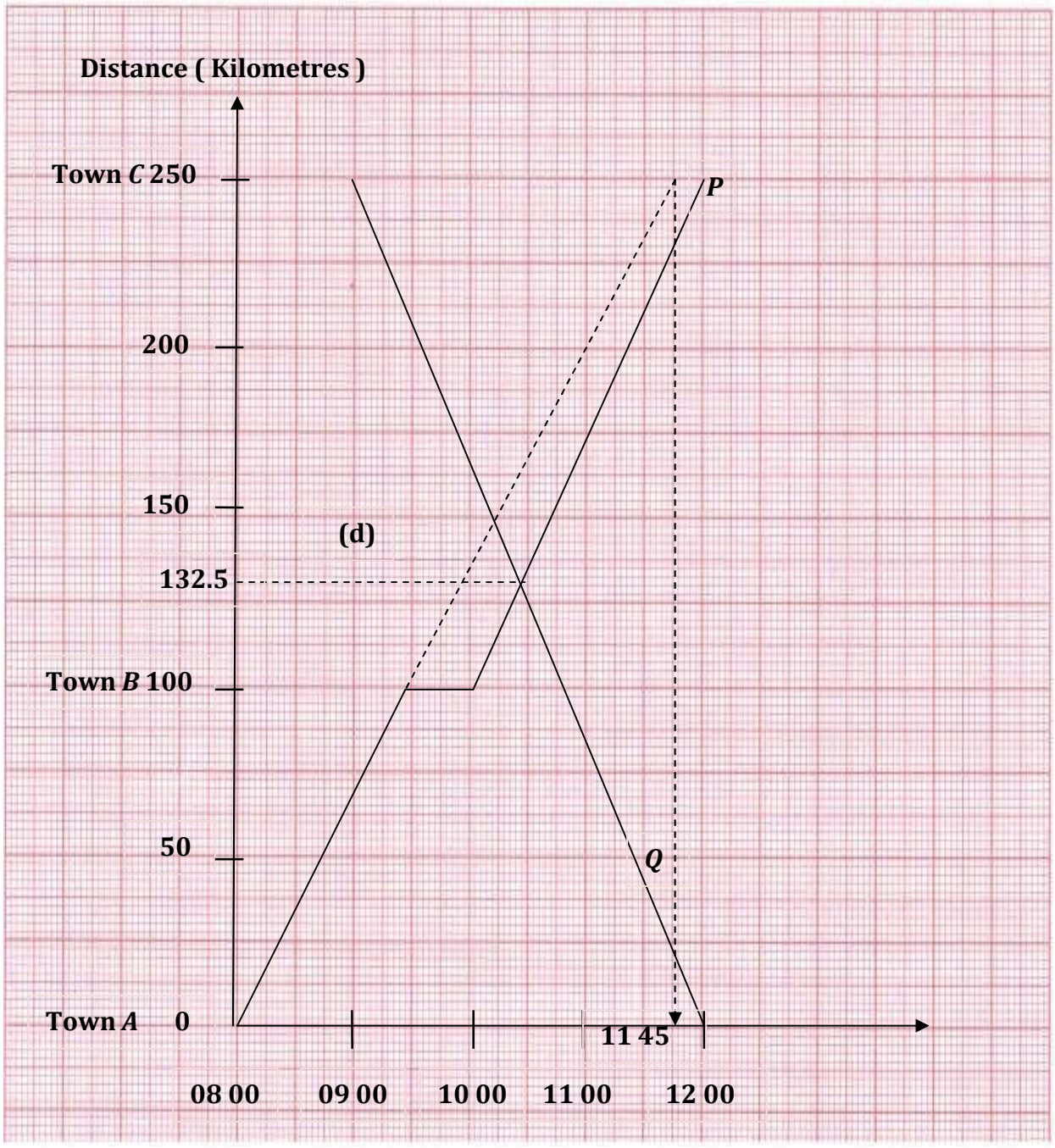
c) **The average speed of Train *Q* for the whole journey ,**

d) **The distance from Town *A* when the trains cross each other ,**

e) **The time P would have reached Town *C* if it did not stop at Town *B* .**


Distance (Kilometres)
Town C 250
200
150
Town B 100
50
Town A 0
P
Q
08 00
09 00
10 00
11 00
12 00

SOLUTION :



a) i) Speed of Train *P* (From 08 00 to 09 30)

$= \frac{100\ km}{1\frac{1}{2}h}$ ← The gradient of the first part of the graph From 08 00 to 09 30 gives the speed at the time .

$= 60\frac{2}{3}$ km / h

ii) Speed of Train *P* (From 09 30 to 10 00)

= 0 km / h ← The gradient is zero as the graph is a horizontal line From 09 30 to 10 00.

iii) Speed of Train *P* (From 10 00 to 12 00)

$= \frac{(250-100)\text{km}}{2\text{ h}}$ ← The gradient of the third part of the graph From 10 00 to 12 00.

$= \frac{15\ km}{2\ h}$

= 75 km / h

b) Average speed of Train *P* for the whole journey

$= \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$= \frac{250\ km}{4\ h}$

$= = 62\frac{1}{2}$ km / h

c) Average speed of Train ***P*** for the whole journey

$= \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$= \frac{250\ km}{3\ h}$

$= = 83\frac{1}{3}$ km / h

d) The trains cross each other when they are 132.5 km from Town ***A*** .

TIPS FOR STUDENTS:

The trains cross each other when their distance – time graph intersect .

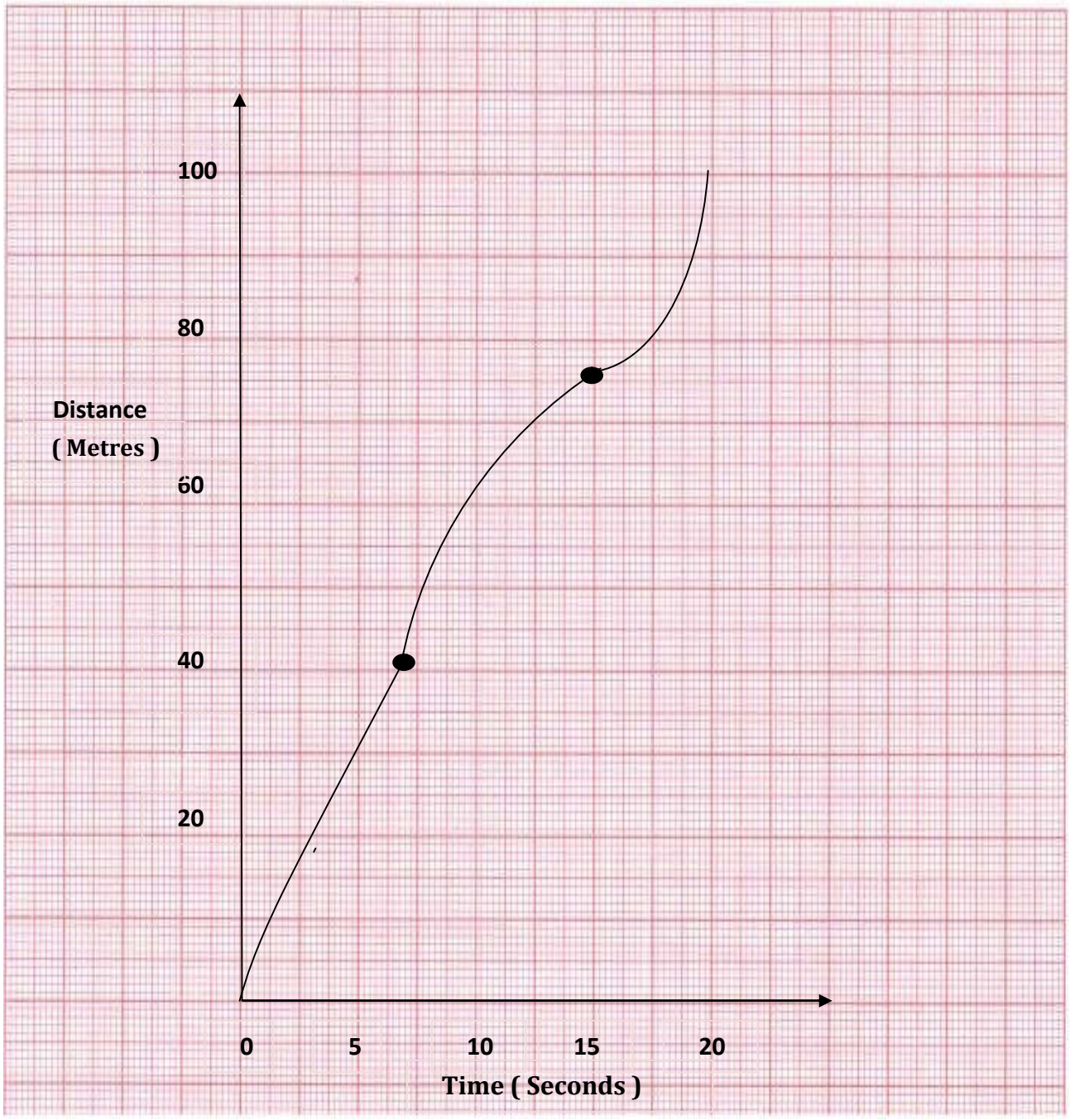
e) Train ***P*** would have reached Town ***C*** at 11 : 45 if it did not stop at Town ***B*** .

EXAMPLE3:

The graph shows how Mailing swum in a 100 m freestyle swimming event. Find her speed in meters per second when.

a) $t = 7$,

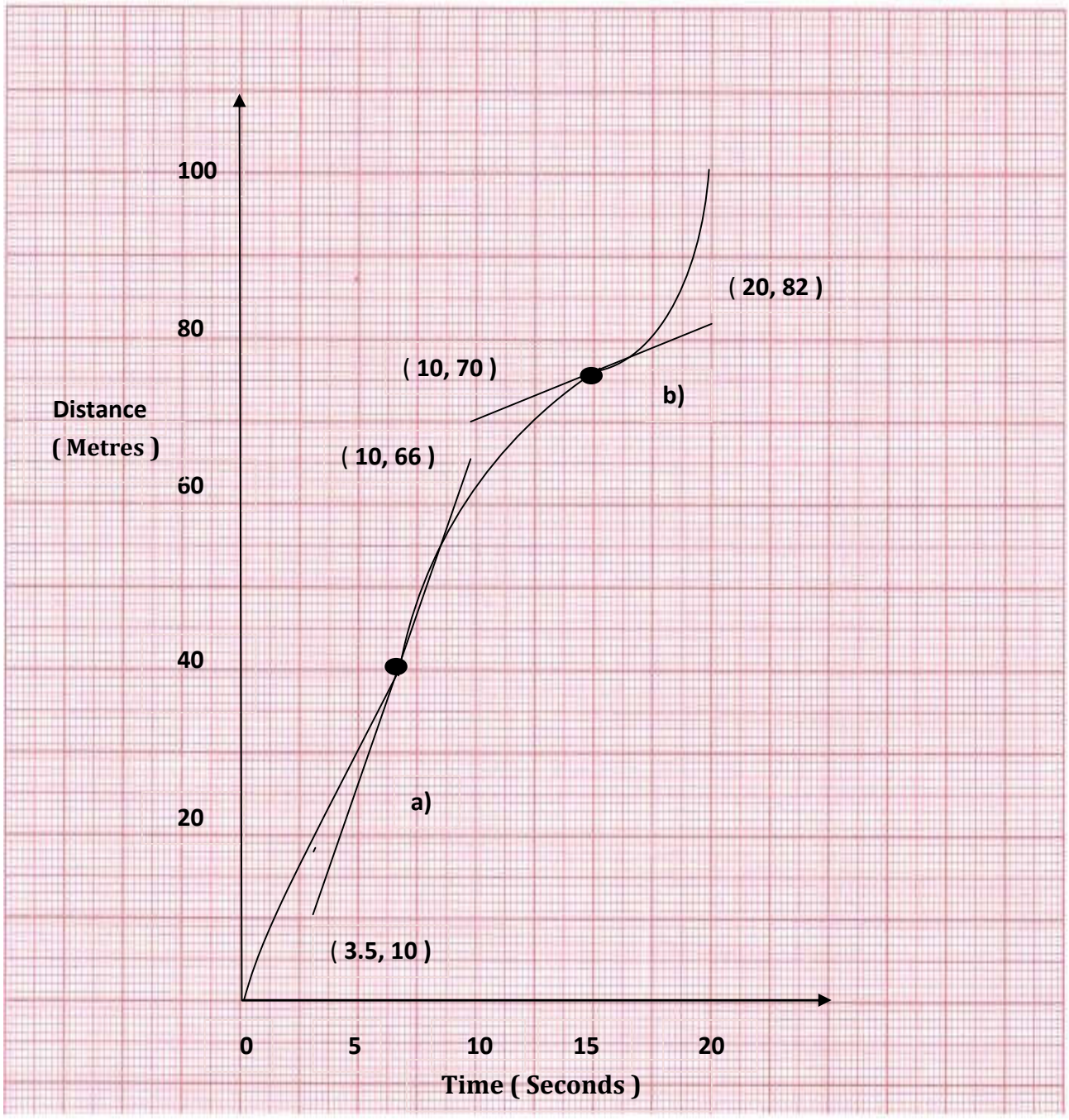
b) $t = 15$.



SOLUTION:

Draw the tangent to the curve at each point to find the gradient .

b) $t = 15$ **.**



a) Speed when $t = 7$

$$= \frac{(66 - 10)\ m}{(10 - 3.5)\ s}$$

$= \frac{56\ km}{6.5\ S}$

$= 8.62$ m / s (Correct to 3 sig . fig .)

b) Speed when $t = 15$

$= \frac{(82 - 70)\ m}{(20 - 10)\ s}$

$= \frac{12\ m}{10\ s}$

$= 1.2$ m / s

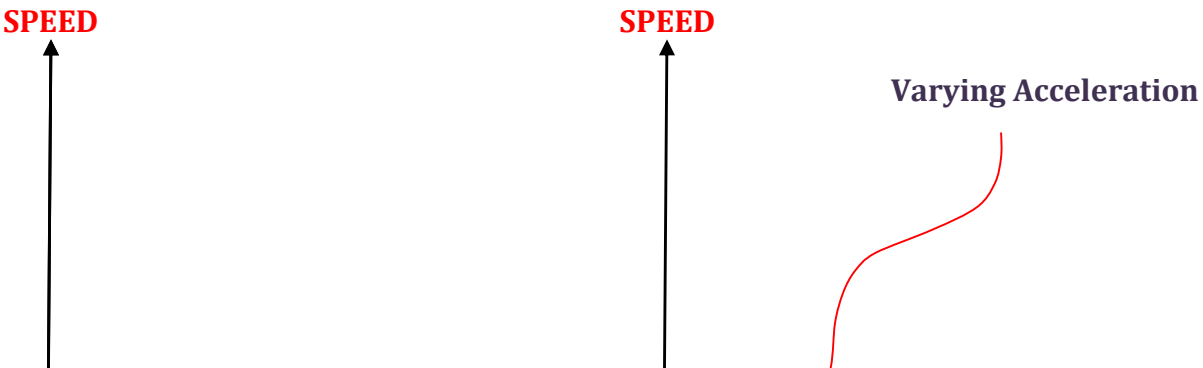
SPEED – TIME GRAPHS:

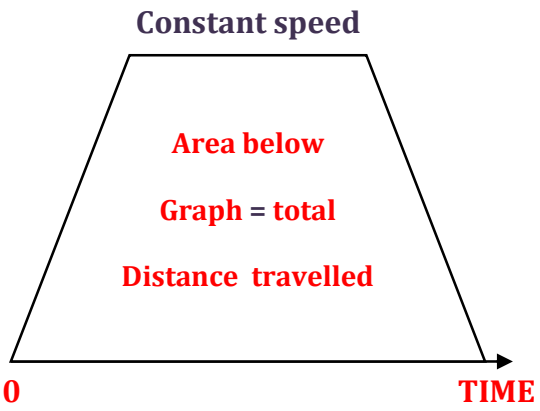
a) The gradient of a speed – time graph gives the instantaneous acceleration of a moving object .

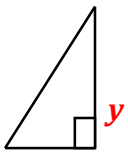
b) A positive gradient represents acceleration .

A positive gradient represents deceleration or retardation .

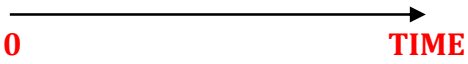
c) The area under a speed – time graph gives the total distance travelled .





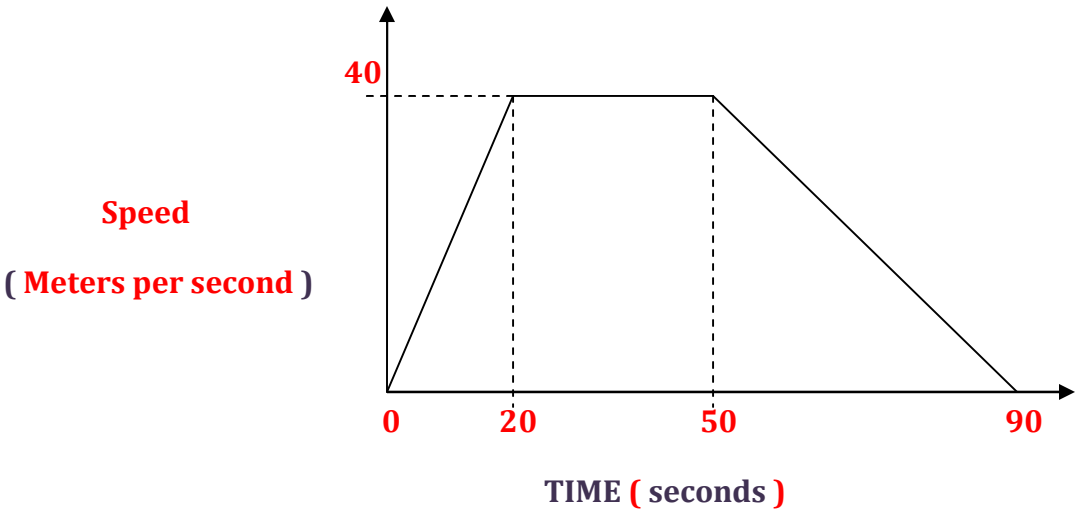


The gradient $\left(\frac{y}{x}\right)$ gives the acceleration of the object



EXAMPLE1:

The diagram shows the speed – time graph of a particular journey .



a) Find the acceleration

i) In the first 20 seconds,

ii)Between 20 and 50 seconds,

iii) In the last 40 seconds.

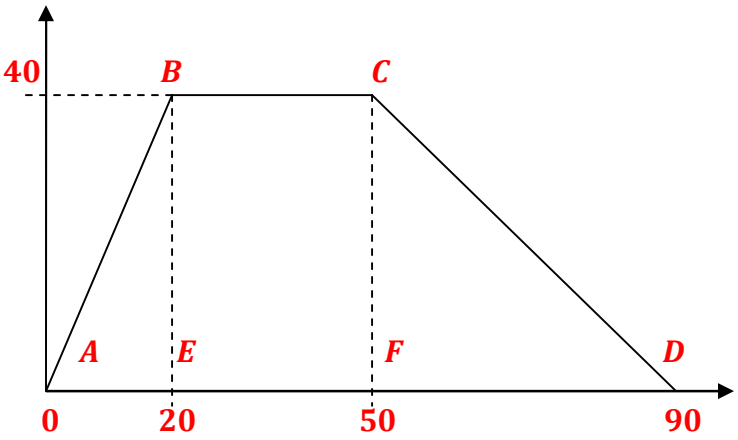
b)Calculate

i) The total distance travelled,

ii) The average speed for the whole journey.

c) Sketch the corresponding distance – time graph.

SOLUTION:



a) i) Acceleration in the first 20 seconds

The find the acceleration find the gradient of the graph at the period . The gradient of the line *AB* gives the acceleration .

$= \frac{(40 - 0)\,m}{(20 - 0)\,s}$

$= \frac{40\,m/s}{20\,s}$

$= 2\ m/s^2$

ii) Acceleration between 20 and 50 seconds ,

$= 0\ m/s^2$ ← The gradient of line *BC* is zero .

iii) Acceleration in the last 40 seconds ← The gradient of line *CD* gives the acceleration at the period .

$= \frac{(0 - 40)\,m}{(90 - 50)\,s}$

$= \frac{-40\,m/s}{-40\,s}$

$= -1\ m/s^2$

TIPS FOR STUDENTS:

The negative acceleration, called retardation or deceleration shows that the object is slowing down .

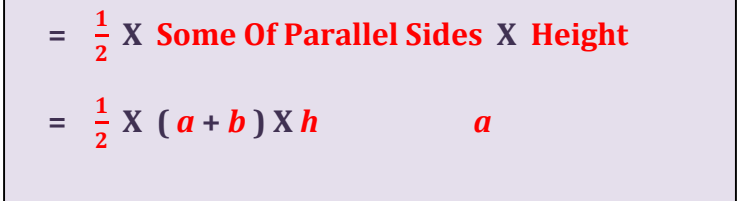
b) i) Total distance travelled

$= \frac{1}{2} \times (AD + BC) \times BE$ ←

Area Of Trapezium

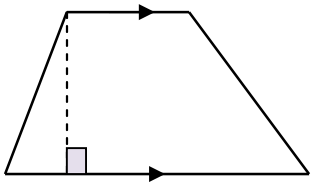
$= \frac{1}{2}$ X Some Of Parallel Sides X Height

$= \frac{1}{2} \times (a + b) \times h$



$= \frac{1}{2} \times (90 + 30) \times 40$

$= 2400 \text{ m}$



TIPS FOR STUDENTS:

To find the total distance travelled, find the area under the graph, i.e. find the area of trapezium *ABCD* .

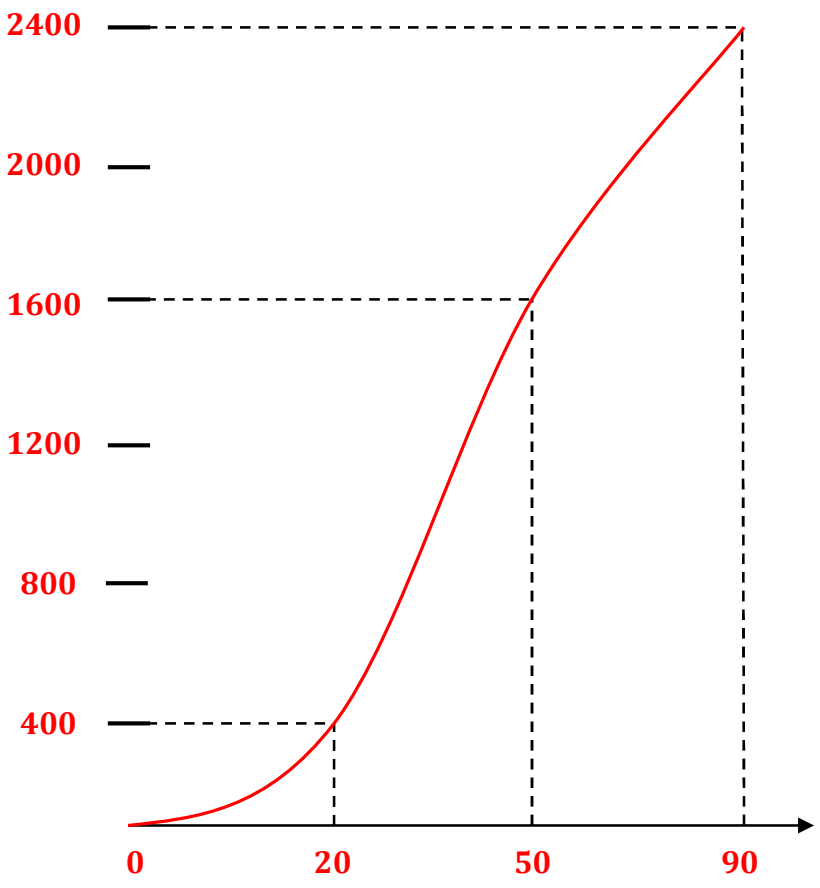
ii) Average speed for the whole journey.

$= \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$= \frac{2400\ m}{90\ s}$

$= 26\frac{2}{3} \text{ m / s}$

c)



Distance travelled (First 20 Seconds)

$= \frac{1}{2} \times AE \times BE$

$= \frac{1}{2} \times 20 \times 40$

$= 400$ m

Distance travelled (Between 20 and 50 seconds)

$= BE \times BC$

= 40 X 30

= 1200 m

Distance travelled (Last 40 Seconds)

= $\frac{1}{2}$ X *FD* X *CF*

= $\frac{1}{2}$ X 40 X 40

= 800 m

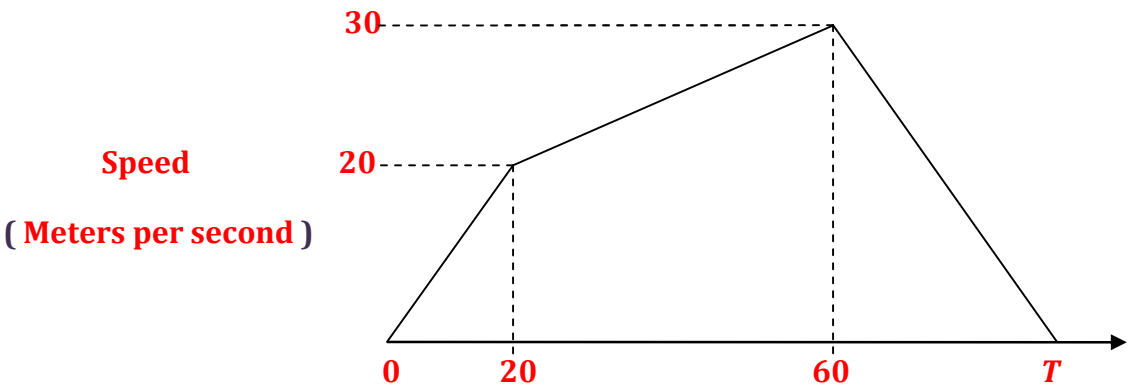
TIPS FOR STUDENTS:

IN THE DISTANCE – TIME GRAPH,

- ✓ The First part of the graph, (0, 0) to (20, 400) is a curve . This is because the speed is not uniform .
- ✓ The Second part of the graph, (20, 400) to (50,1600) is a curve . This is because of uniform .
- ✓ The Third part of the graph, (50, 1600) to (90, 2400) is a curve. This is because the speed is not uniform .

EXAMPLE2:

The diagram shows the speed – time graph for *T* seconds of the motion of an object .



a) Calculate

i) The speed of the object when $t = 30$,

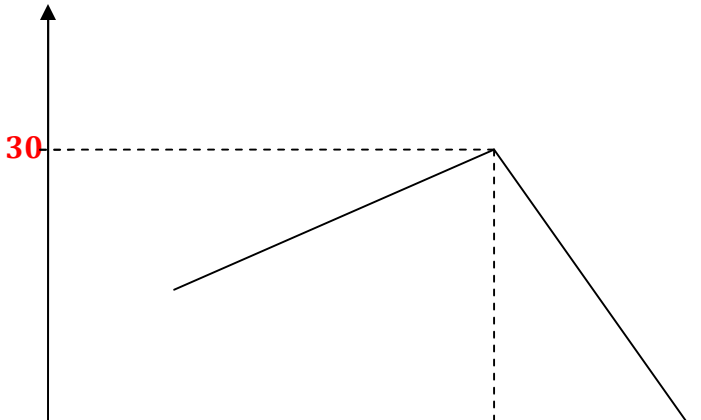
ii) The acceleration of the object when $t = 30$,

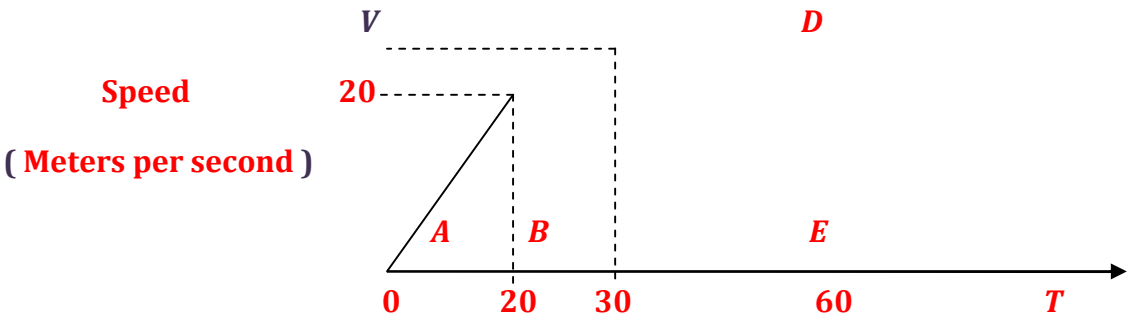
iii) The distance travelled in the first 60 seconds.

b) Give that the distance travelled is 1575 m, find the value of T.

c) Find the retardation when $t = 70$.

SOLUTION:





a) i) Let the speed when $t = 30$ be v m/s .

$$= \frac{v - 20}{30 - 20} = \frac{30 - 20}{60 - 20}$$

The acceleration is constant $t = 30$ to $t = 30$.

$$= \frac{v - 20}{10} = \frac{10}{40}$$

$$V - 20 = \frac{1}{4} \times 10$$

$$V - 20 = 2.5$$

$$V = 22.5 \text{ m / s}$$

$\therefore$ The speed when $t = 30$ is 22.5 m / s

ii) Acceleration when $t = 30$

$$= \frac{(30 - 20)m/s}{(60 - 20)s}$$

$= \frac{10\ m/s}{40\ s}$

$= \frac{1}{4}\ m/s^2$

iii) Distance travelled in the first 60 seconds

= Area of ΔABC + Area of trapezium $BCDE$

$= \frac{1}{2} \times 20 \times 20 + \frac{1}{2} \times (20 + 30) \times 40$

$= 200 + 1000$

$= 1200$ m

The area under a speed – time graph gives the distance travelled during that period .

b) Total distance travelled = 1575 m

$1200 + \frac{1}{2} \times (T - 60) \times 30 = 1575$ m

$1200 + 15(T - 60) = 1575$

$T - 60 = 25$

$T = 25$

Divide both sides by 15 .

c) Acceleration when $t = 70$

$= \frac{(0 - 30)m/s}{(85 - 60)\ s}$

The gradient of the graph from $t = 60$ to $t = 85$ gives the acceleration when $t = 70$.

$= \frac{-30\ m/s}{25\ s}$

$= 1.2\ m/s^2$

$\therefore$ The retardation when $t = 70$ is $1.2\ m/s^2$

EXAMPLE3:

The diagram shows the speed – time graph of the first 30 seconds of a journey .

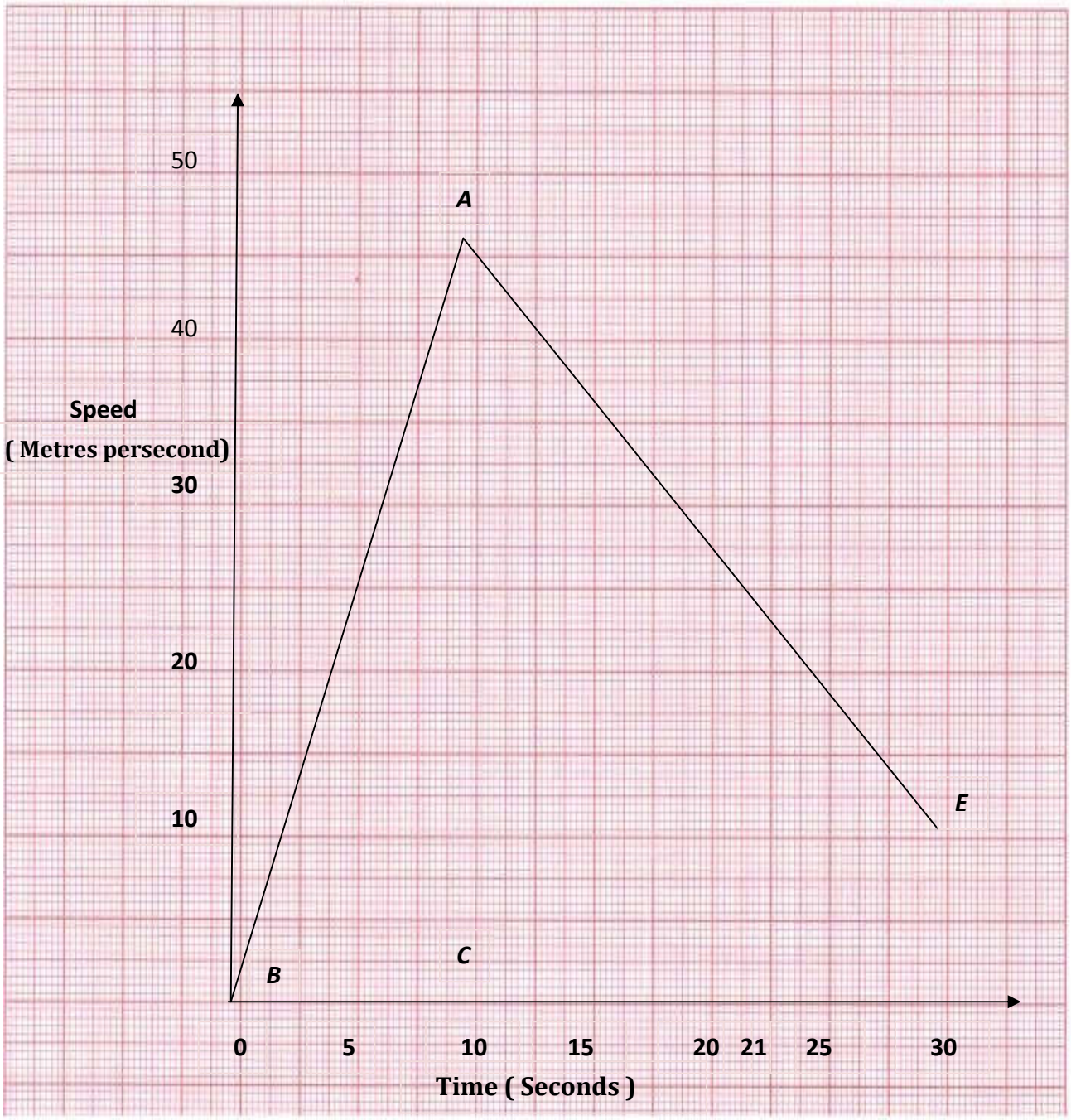
FIND

a) The speed when $t = 21$,

b) The acceleration during the first 10 seconds,

c) The total distance in the 30 seconds.

SOLUTION:


50
40
Speed
(Metres persecond)
30
20
10
A
E
C
B
0
5
10
15
20
21
25
30
Time (Seconds)

a) From the graph, when $t = 21$, speed = 26 m/s .

b) Acceleration during the first 10 seconds ,

$= \frac{(46 - 0)m/s}{(10 - 0)s}$ ← The gradient of the line AB gives the acceleration .

$= \frac{46\ m/s}{10\ s}$

$= 4.6\ m/s^2$

c) Total distance travelled

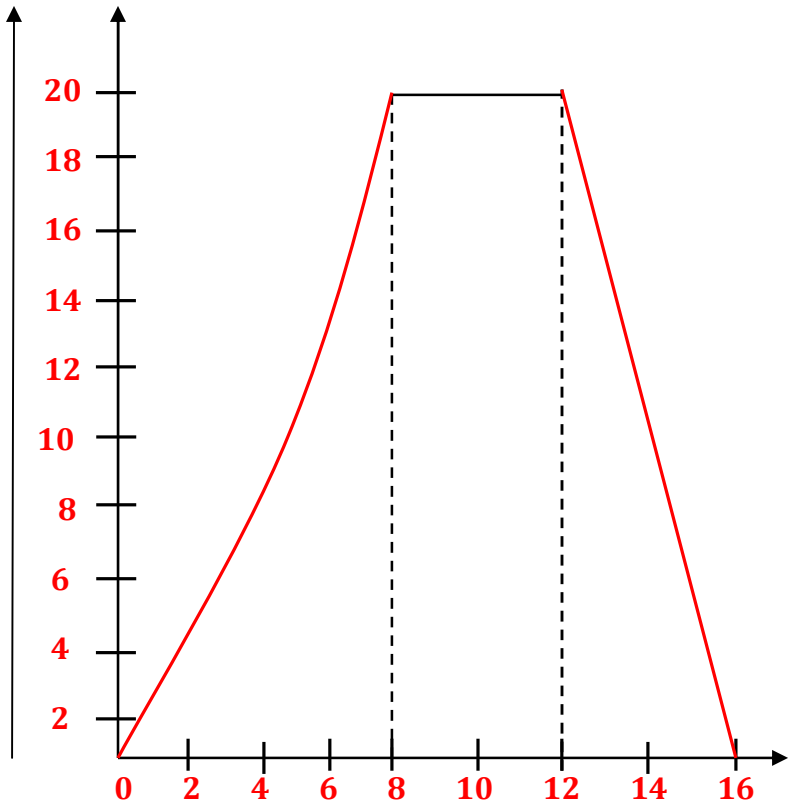
= Area of ΔABC + Area of trapezium $ACDE$

$= \frac{1}{2} \times 10 \times 46 + \frac{1}{2} \times (46 + 10) \times 20$

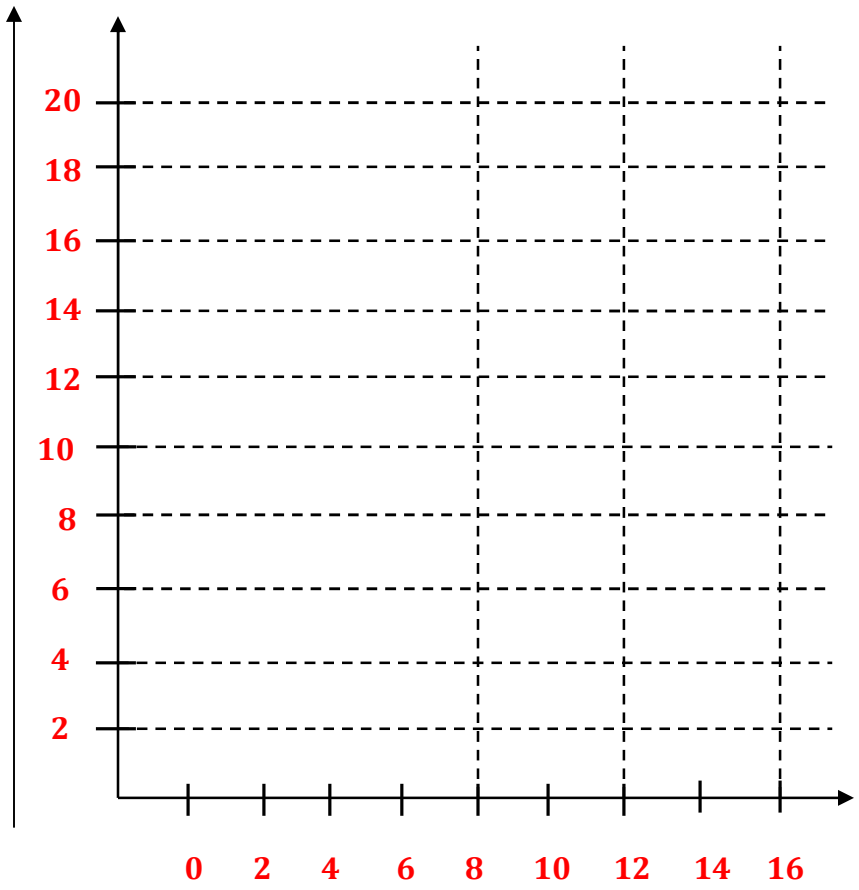
$= 230 + 560$

$= 790$ m

EXAMPLE4:


DISTANCE (meters)
20
18
16
14
12
10
8
6
4
2
0
2
4
6
8
10
12
14
16
TIME (t second)

SPEED (Meters per second)



TIME (*t* second)

Diagram A shows the distance – time graph for the first 16 seconds of a journey .

a) Find the average speed during the first 8 seconds .

b) FIND

i) The speed,

ii) The acceleration,

During the last **4** second .

c) The speed increases uniformly from **6** m/s when t = **0** to **10** m/s when t = **8** . Sketch the speed – time graph for the same journey in diagram ***B*** .

SOLUTION:

a) Average speed = $\frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$= \frac{20\ m}{8\ s}$

$= 2.5$ m / s

b) **i)** Speed = $\frac{20\ m}{4\ s}$

$= 5$ m/s

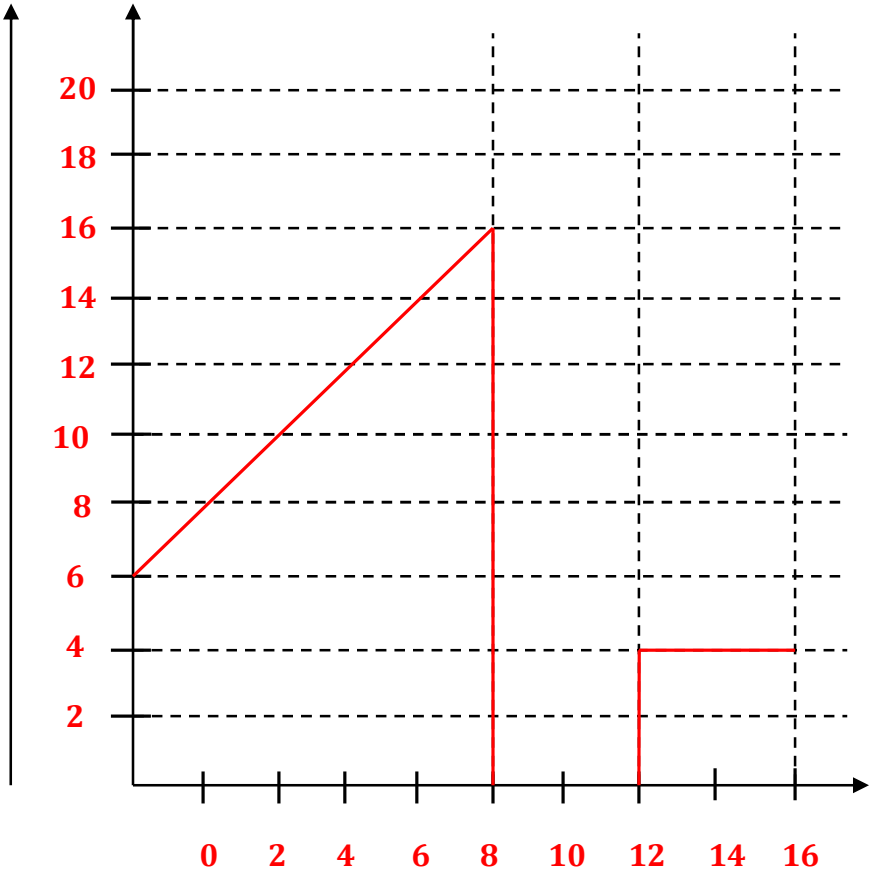
ii) Acceleration = **0** m/s^2

TIPS FOR STUDENTS:

Acceleration is zero since speed is constant.

c)

SPEED (Meters per second)



TIME (*t* second)

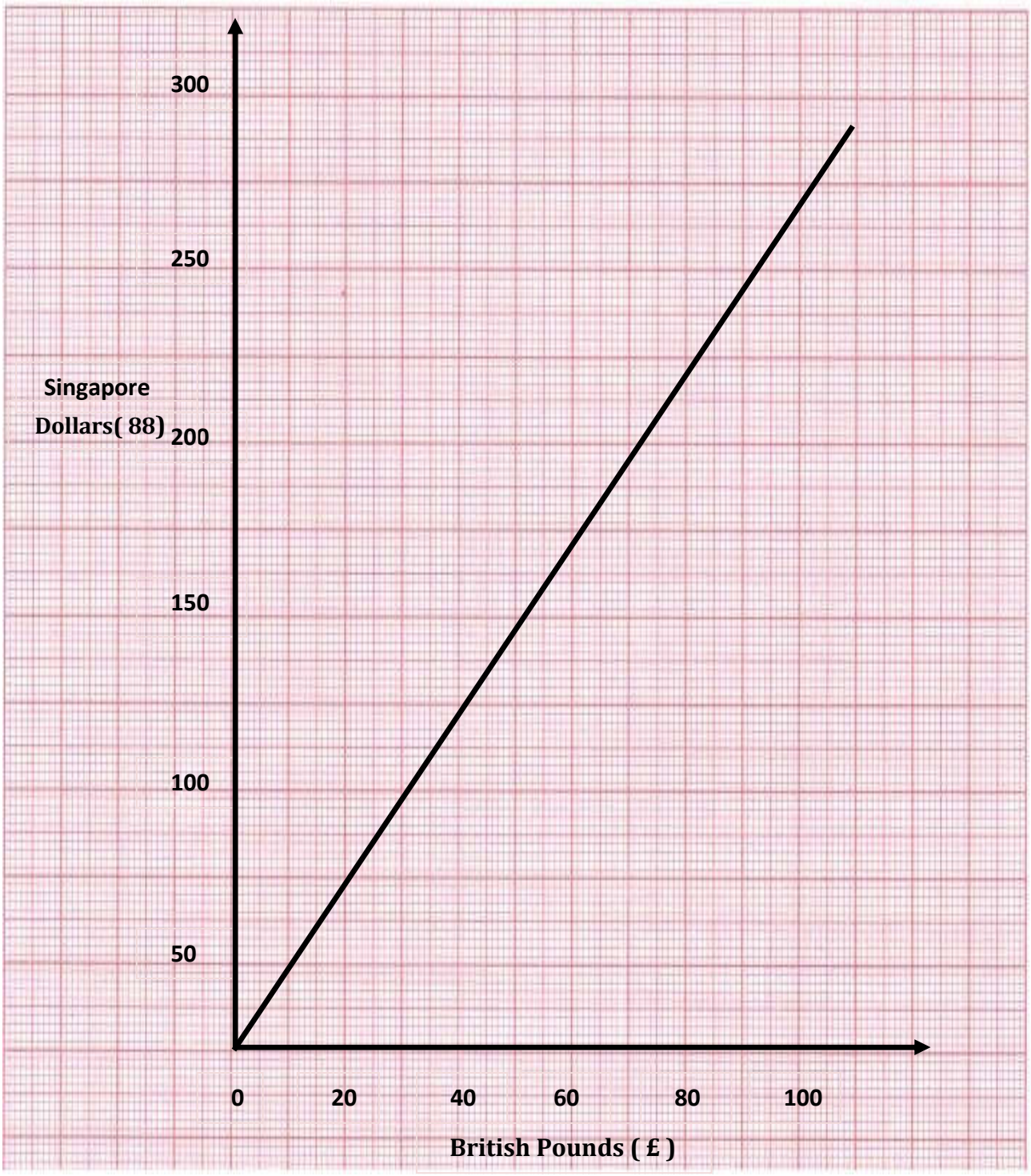
1.2 CONVERSION GRAPHS:

A Conversion graph is used to convert one quantity to the other quantity.

e.g.- £ to US Dollers.

EXAMPLE:

The diagram shows a conversion graph between Singapore dollars (S $) and the British pounds (£).



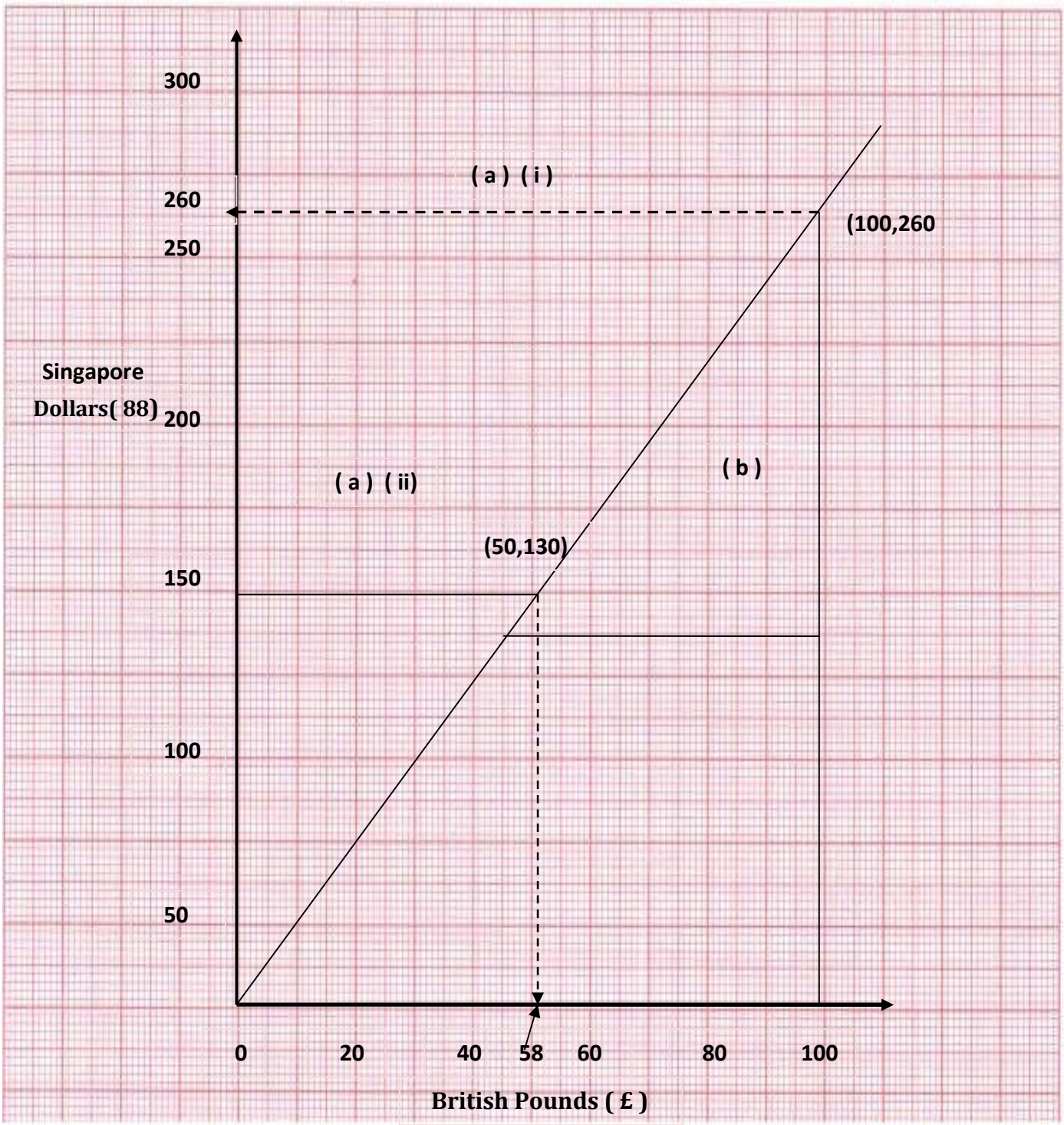
a) Use the graph to convert

i) £ 100 to Singapore $,

ii) S $ 150 to British £.

b) Approximately how many Singapore dollars are equivalent to £ 1?

SOLUTION:


300
260
250
200
150
100
50
Singapore
Dollars(88)
(a) (i)
(100,260
(a) (ii)
(b)
(50,130)
0
20
40
58
60
80
100
British Pounds (£)

a) From the graph,

i) £ 100 ≈ S $ 260,

ii) S $ 150 ≈ £ 58 .

b) Gradient = $\frac{260 - 130}{100 - 50}$

= $\frac{130}{50}$

= 2.6

∴ £ 1 ≈ S $ 2.60

The gradient gives the exchange rate of S $ 2.60 to each £ 1.

SUMMARY AND KEY POINTS

1.) The gradient of a distance – time graph gives the instantaneous speed of a moving object .

2.) AVERAGE SPEED = $\frac{Total\ Distance\ Travelled}{Total\ Time\ Taken}$

3.) SPEED – TIME GRAPHS:

a) The gradient of a speed – time graph gives the instantaneous acceleration of a moving object .

b) A positive gradient represents acceleration.

A positive gradient represents deceleration or retardation.

c) The area under a speed – time graph gives the total distance travelled.

4.) The negative acceleration, called retardation or deceleration shows that the object is slowing down .

5.) Average speed for the whole journey = $\frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

6.) Acceleration is zero if speed is constant during distance travelled.

7.) A Conversion graph is used to convert one quantity to the other quantity

e.g. £ to US Dollars.