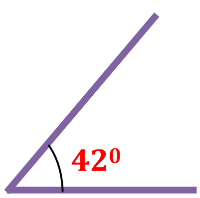
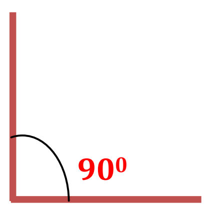
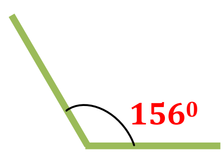
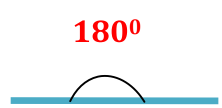
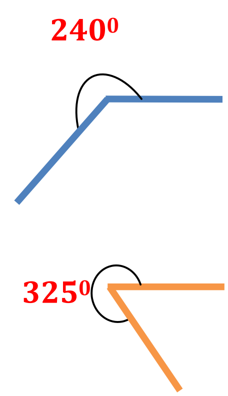
GEOMETRICAL PROPERTIES OF ANGLES AND CIRCLES, ANGLES PROPERTIES OF TRIANGLES, QUADRILATERALS AND POLYGONS:

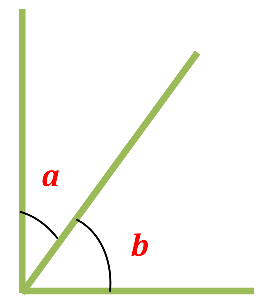
1.1 TYPES OF ANGLES:

ACUTE ANGLE	RIGHT ANGLE	OBTUSE ANGLE	STRAIGHT ANGLE	REFLEX ANGLE
				
Angles less than 90^0.	**Angles equal to 90^0.**	**Angles greater than 90^0 but less than 180^0.**	**Angles equal to 180^0.**	**Angles greater than 180^0 but less than 360^0.**

1.2 GEOMETRICAL PROPERTIES OF ANGLES:

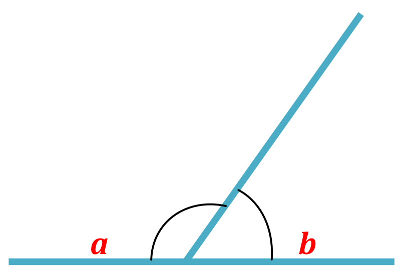
1.COMPLEMENTARY ANGLES: Two angles are complementary if their sum add up to 90^0 .

$a + b = 90^0$



2. SUPPLEMENTARY ANGLES: Two angles are supplementary if their sum add up to 180^0 .

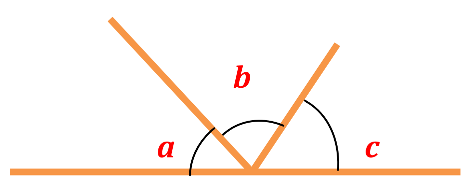
$a + b = 180^0$



3. The sum of ADJACENT ANGLES on a straight line is equal to 180^0.

$a + b + c = 180^0$

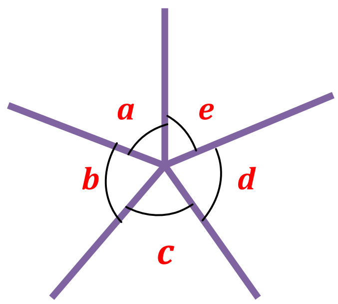
(adj . ∠s on a str . line .)



4. Angles at a point add up to 360^0.

$$a + b + c + d + e = 360^0$$

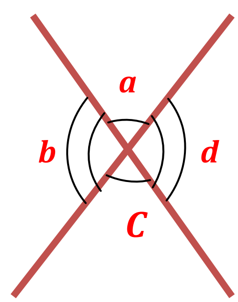
(∠s, at a point .)



5. Vertically opposite angles are equal.

$$a = c$$
$$b = d$$

(vert . opp . ∠s.)

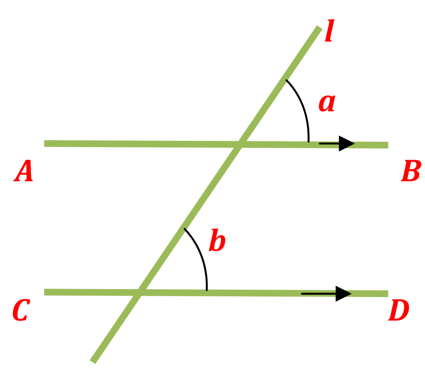


6. Angles formed by parallel lines cut by a transversal, *l* .

a) CORRESPONDING ANGLES are equal.

$$a = b$$

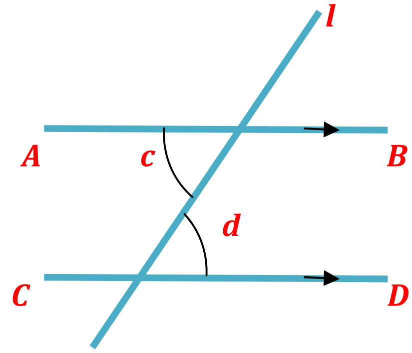
(corr. ∠s, *AB* // *CD*)



b) **ALTERNATE ANGLES are equal.**

$c = d$

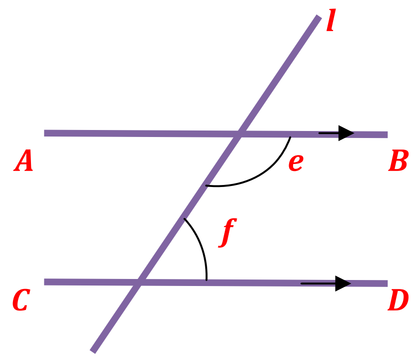
(alt. ∠s, $AB // CD$)



c) **INTERIOR ANGLES are supplementary.**

$e = f = 180^0$

(int. ∠s, $AB // CD$)



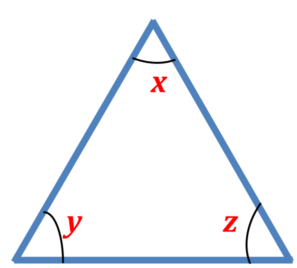
1.3 ANGLES PROPERTIES OF TRIANGLES AND QUADRILATERALS:

ANGLE PROPERTIES OF TRIANGLES:

1. The sum of the 3 angles of a triangle is equal to 180^0.

$x + y + z = 180^0$

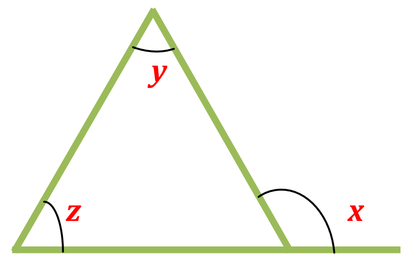
(∠ sum of Δ)



2. The exterior angle of a triangle is equal to the sum of the interior opposite angles.

$$x = y + z$$

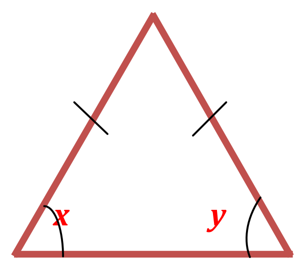
(ext . ∠ of Δ)



3. An Isosceles Triangle has 2 equal angles opposite the 2 equal sides.

$$x = y$$

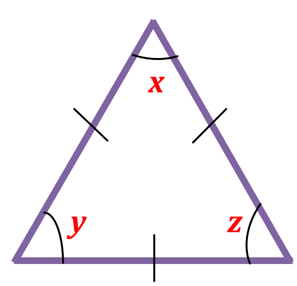
(base ∠s of isos . Δ)



4. An Equilateral Triangle has 3 equal sides and 3 equal angles, each equal to 60^0 .

$$x + y + z = 60^0$$

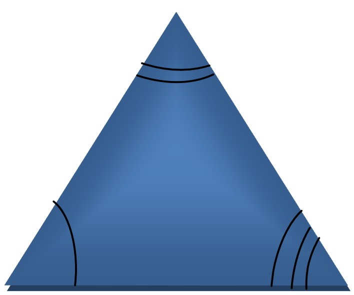
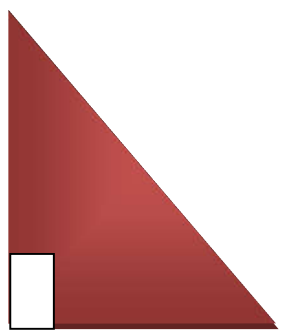
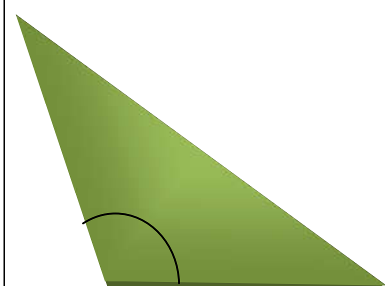
(∠ of equi . Δ)



TIPS FOR STUDENTS :

A **Scalene Triangle** has no equal sides and all the angles are different in size.

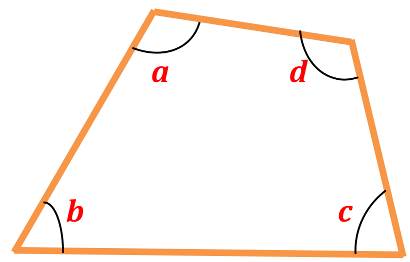
5. A triangle can be grouped according to the types of angles it contains as shown in the table below .

ACTUE – ANGLED TRIANGLE	RIGHT – ANGLED TRIANGLE	OBTUSE – ANGLED TRIANGLE
		
All Three Angles Are Acute.	One Right Angle .	One Obtuse Angle .

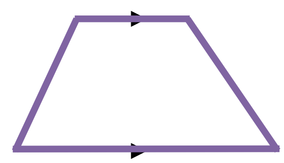
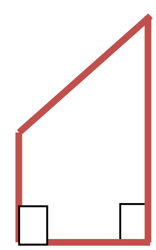
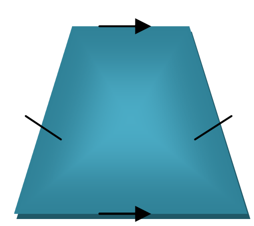
ANGLE PROPERTIES OF QUADRILATERALS:

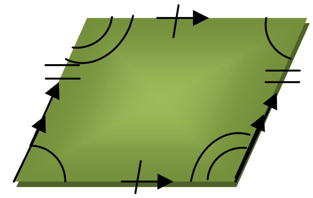
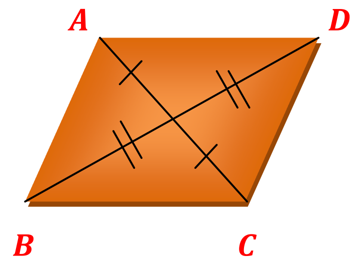
1. The sum of all the angles in a quadrilateral is 360^0.

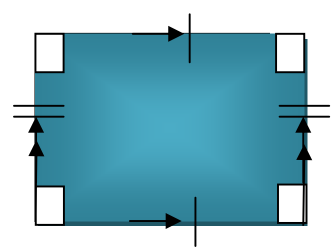
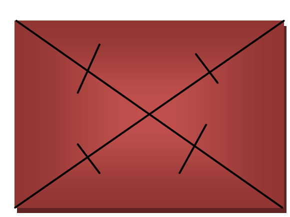
$a + b + c + d$	($\angle$ sum of quad .)

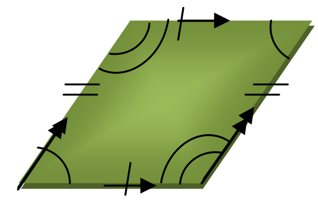
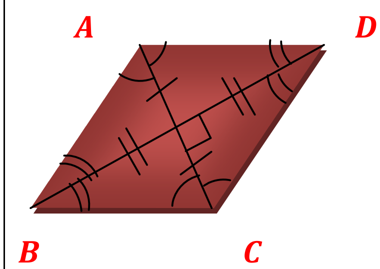


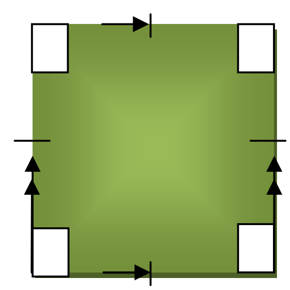
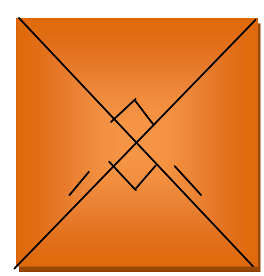
2. **The properties of some special quadrilaterals are given in the table below.**

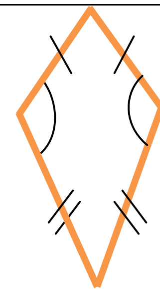
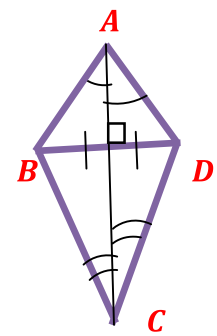
NAME	DESCRIPTION	EXAMPLE
TRAPEZIUM	➢ **One pair of parallel opposite sides.**	 
ISOSCELES TRAPEZIUM	➢ **One pair of parallel opposite sides** ➢ **Non – parallel sides are equal in length.**	

PARALLELOGRAM	➢ Two pairs of parallel opposite sides . ➢ Opposite sides are equal in length . ➢ Opposite angles are equal . ➢ Diagonals bisects each other.	  *AX* = *XC* *BX* = *XD*

NAME	DESCRIPTION	EXAMPLE
RECTANGLE	➢ Two pairs of parallel opposite sides . ➢ Opposite sides are equal in length . ➢ All four angles are right angles . (90^0) ➢ Diagonals are equal in length . ➢ Diagonals bisect each other .	 

RHOMBUS	➢ **Two pairs of parallel opposite sides.** ➢ **Four equal sides.** ➢ **Opposite angles are equal.** ➢ **Diagonals bisect each other at right angles.** ➢ **Diagonals bisect the interior angles.**	  ***AX = XC*** ***BX = XD***

NAME	**DESCRIPTION**	**EXAMPLE**
SQUARE	➢ **Two pairs of parallel opposite sides.** ➢ **Four equal sides.** ➢ **All four angles are right angles. (90^0)** ➢ **Diagonals are equal in length.** ➢ **Diagonals bisect each other at right angles .** ➢ **Diagonals bisect the**	 

	interior angles.	
KITE	➢ No parallel sides. ➢ Two pairs of equal adjacent sides. ➢ One pair of equal opposite angles. ➢ Diagonals intersect at right angles . ➢ One diagonals bisect the interior angles .	  $\angle BAC = \angle DAC$ $\angle BCA = \angle DCA$

TIPS FOR STUDENTS:

A rectangle with 4 equal sides is a square.

A Parallelogram with 4 right angles is a rectangle.

A parallelogram with 4 equal sides is a rhombus.

A rhombus with 4 equal angles is a square.

EXAMPLE1:

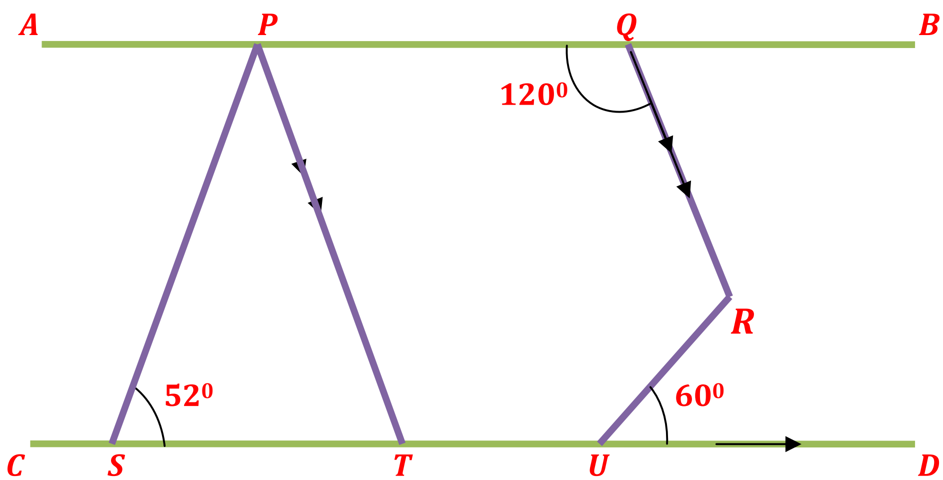
In the diagram, AB is parallel to CD and PT is parallel to QR . Given that $\angle PST = 52^0$, $\angle PQR = 120^0$ and $\angle RUD = 60^0$.

Find

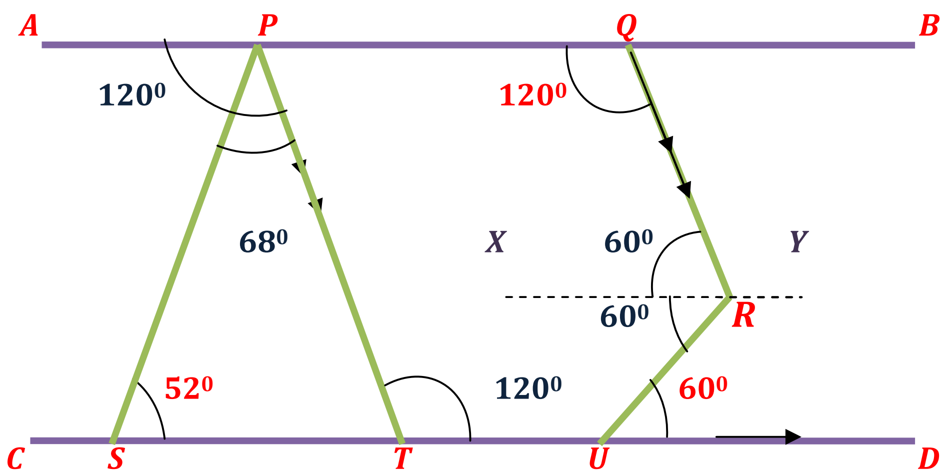
a) $\angle PTU$,

b) $\angle SPT$,

c) $\angle QRU$.



SOLUTION:



a) $\angle PTU = \angle PQR$ (corr, $\angle$ s, $PT // QR$)

$= 120^0$

$\angle PTU = \angle APT$ (alt, $\angle$ s, $AB // CD$)

$= 120^0$

b) $\angle SPT + \angle PST = \angle PTU$ (ext, $\angle$ s of Δ)

$\angle SPT + 52^0 = 120^0$

$= 120^0 - 52^0$

$= 68^0$

c) Draw the line XY through R which is parallel to AB .

$\angle QRX + \angle PQR = 180^0$ (aint, $\angle$ s, $AB // XY$)

$\angle SPT + 120^0 = 180^0$

$\angle QRX = 60^0$

$\angle XRU = \angle RUD$ (alt, $\angle$ s, $XY // CD$)

$= 60^0$

$\therefore \angle QRU = \angle QRX + \angle XRU$

$= 60^0 + 60^0$

$= 120^0$

EXAMPLE2 :

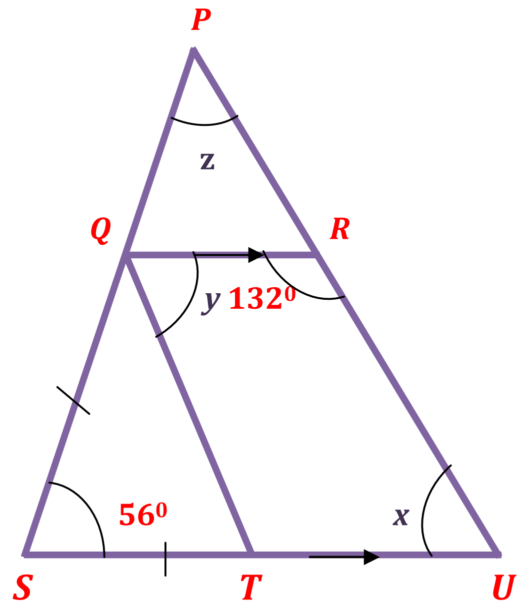
In the diagram, ***PQR*** is a triangle. The point ***S*** is on ***PQ*** producer, the point ***U*** is on ***PR*** produced and ***STU*** is a straight line . ***SQ*** = ***ST*** and ***QR*** is parallel to ***SU*** . Given that $\angle QRU = 132^0$ and $\angle QST = 56^0$.

CALCULATE

a) x ,

b) y ,

c) z .



SOLUTION:

a) $x + 132^0 = 180^0$ (int, $\angle$ s, QR // SU)

$\therefore x = 180^0 - 132^0$

$= 48^0$

b) $\angle STQ = \frac{180^0 - 56^0}{2}$ (base, $\angle$ s of isos . Δ)

$= 62^0$

$\therefore\ y = \angle STQ$ (alt, $\angle$s, $QR \,//\, SU$)

$= 62^0$

c) $z + 56^0 + x = 180^0$ (sum of Δ)

$z + 56^0 + 48^0 = 180^0$

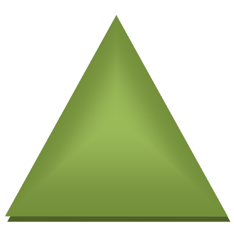
$\therefore\ y = 180^0 + 104^0$

$= 76^0$

1.4 ANGLE OF PROPERTIES OF POLYGONS:

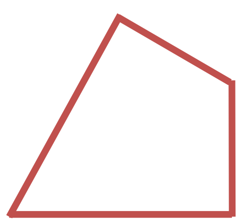
1. A polygon is a closed plane figure with three or more straight lines.

e.g.



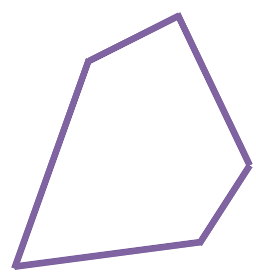
3 SIDES

TRIANGLE



4 SIDES

QUADRILATERAL



5 SIDES

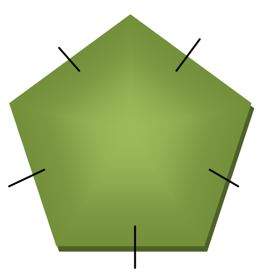
PENTAGON

2. Polygons are named according to the number of sides they have.

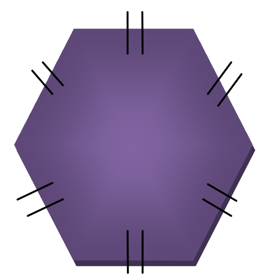
The table below shows the names of some polygons.

NUMBER OF SIDES	NAME OF POLYGON
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

3. In a **regular** polygon, the sides are all equal in length and the interior angles are equal .



REGULAR PENTAGON



REGULAR HEXAGON

TIPS FOR STUDENTS:

A polygon with *n* sides is called an *n* – *gon* .

e.g. A Polygon with 12 sides is called a 12 – *gon* .

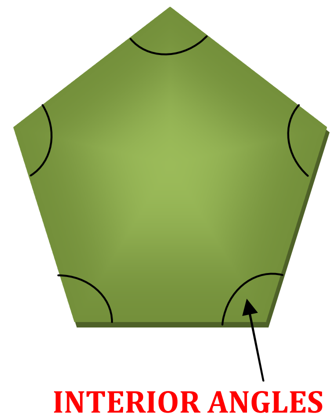
INTERIOR ANGLES OF A POLYGON:

For a n – sided polygon,

Sum of the interior angle = $(n - 2) \times 180^0$

For regular n – sided polygon,

Size of each interior angle = $\frac{(n - 2) \times 180^0}{n}$



EXAMPLE3:

CALCULATE

a) The sum of the interior angles of an octagon ,

b) The size of each interior angles in a regular 18 – sides polygon .

c) The size of each interior angle in regular hexagon .

SOLUTION:

a) Sum of interior angles of an octagon

$= (8 - 2) \times 180^0$

$= 6 \times 180^0$

$= 1080^0$

Sum of each interior $\angle$s of a regular polygon

$= (n - 2) \times 180^0$

An octagon has 8 sides

$n = 8$

b) Size of each interior angle of a regular 18 – sides polygon

$= \frac{(18 - 2) \times 180^0}{18}$

$= \frac{16 \times 180^0}{18}$

$= 160^0$

Sum of the interior $\angle$s of a polygon

$= \frac{(n - 2) \times 180^0}{n}$

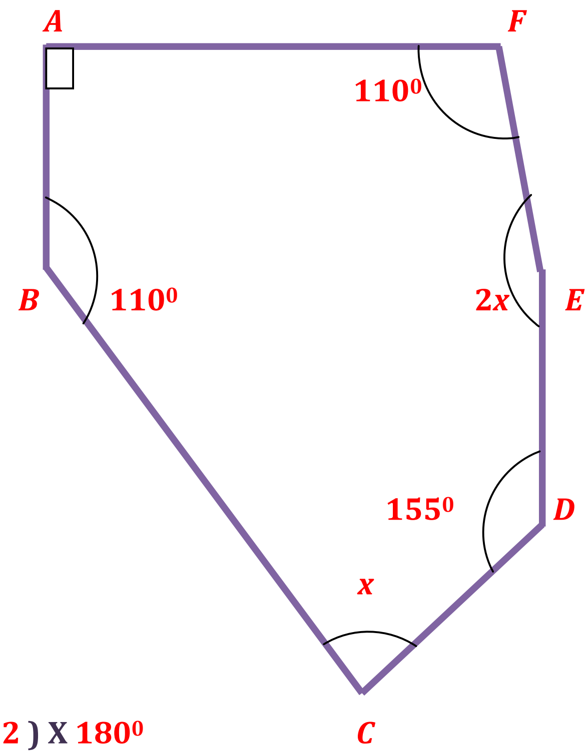
Hence, $n = 18$.

c) Size of each interior angle of a regular hexagon

$= \frac{(6 - 2) \times 180^0}{6} = \frac{4 \times 180^0}{6} = \frac{4 \times 30^0}{6} = 120^0$

EXAMPLE4 :

ABCDEF **is a hexagon . Find the value of** x **.**



SOLUTION:

Sum of interior angles of a hexagon $= (6 - 2) \times 180^0$

$= 720^0$

$90^0 + 140^0 + x + 155^0 + 2x + 110^0 = 720^0$

$3x + 495^0 = 720^0$

$3x = 225^0$

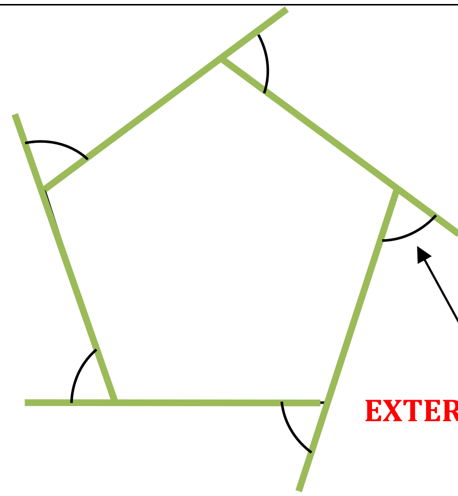
$x = 75^0$

EXTERIOR ANGLES OF A POLYGON:

Sum of all the exterior angles of any polygon $= 360^0$

For a regular **n – sided** polygon,

Size of each exterior angle = $\frac{360^0}{n}$



EXAMPLE 5 :

a) Calculate the size of each exterior angles of a regular pentagon .

b) The exterior of a regular polygon is 24^0 . How many sides does it have ?

SOLUTION:

a) Size of each exterior angles of a regular pentagon

$= \frac{360^0}{n}$

$= 72^0$

Sum of the exterior ∠ s of a regular polygon

$= \frac{360^0}{n}$

A pentagon has 5 sides . $\therefore\ n = 5$

b) Number of sides of the regular polygon

$= \frac{360^0}{24^0}$

$= 15$

EXAMPLE6:

a) Each interior angles of a regular polygon is 120^0 greater than each exterior angles of the polygon . Calculate the number of sides of the polygon .

b) One of the exterior angles of an octagon is 66^0 while the rest of the seven exterior angles are each equal to x^0 . Find the value of x .

SOLUTION :

a) Let the size of each exterior angles be x^0 .

$\therefore$ Size of each interior angle = $x^0 + 120^0$.

$$x + (x + 120^0) = 180^0$$

The sum of the int. ∠ and ext ∠ . is equal to 180^0 .

$$2x = 60^0$$

$$x = 30^0$$

$\therefore$ Size of each exterior angle = 30^0 .

Number of sides of polygon

$= \frac{360^0}{30^0}$

$= 12$

b) Sum of exterior angles of an octagon $= 360^0$

$7x + 66^0 = 360^0$

$7x = 294^0$

$x = 42^0$

The sum of the ext. ∠s of Any polygon is 360^0.

EXAMPLE7 :

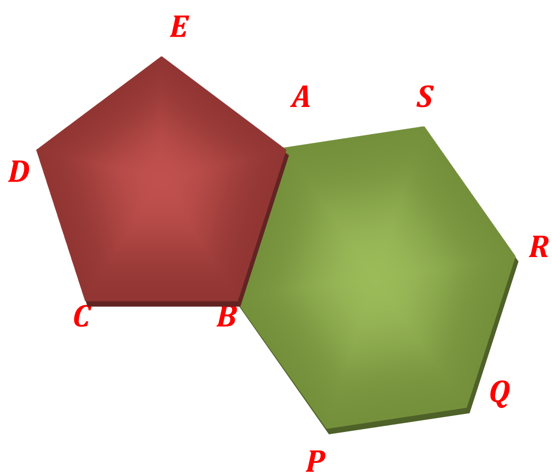
The diagram shows a regular pentagon *ABCDE* and a regular hexagon *ABPQRS* which are drawn on opposite sides of the common line *AB*.

CALCULATE

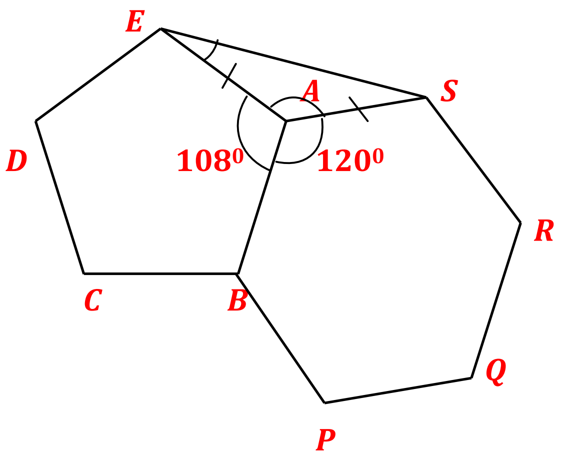
a) ∠*EAB*,

b) ∠*BAS*,

c) ∠*AES*.



SOLUTION:



a) $\angle EAB = \frac{(5-2) \text{ X } 180^0}{5}$

$= 180^0$

Sum of the interior $\angle$s of a regular polygon

$= \frac{(n-2) \text{ X } 180^0}{n}$

Hence, $n = 5$.

b) $\angle BAS = \frac{(6-2) \text{ X } 180^0}{6}$

$= 120^0$

c) $\angle AES = 120^0 - 108^0 - 120^0$ ($\angle$ s at a point)

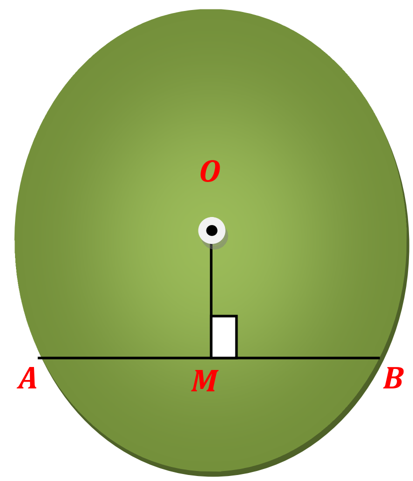
$\angle AES = \frac{180^0 \text{ X } 13\ 2^0}{2}$ (base $\angle$ s of isos . Δ)

$= 24^0$

1.5 SYMMETRICAL PROPERTIES OF CIRCLE:

1. The perpendicular bisects of a chord passes through the centre of the circle . Conversely, a line drawn from the centre of a circle to the midpoint of the chord is perpendicular to chord .

If $OM \perp AB$, then $AM = MB$.

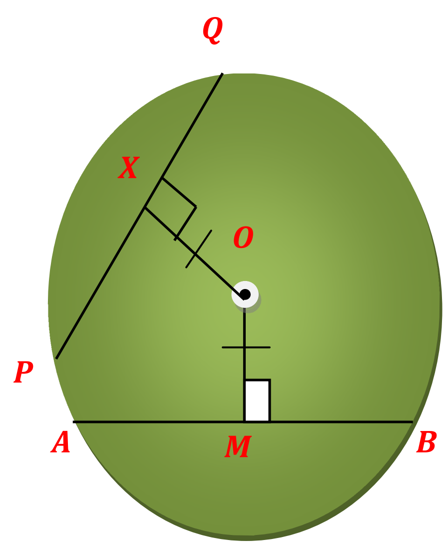


Conversely,

If $AM = MB$, then $OX \perp AB$.

2. Each chords of a circle are equidistant from the centre of the circle . Conversely, chords which are equidistant from the centre of the circle are equal .

If $PQ = RS$, then $OX = OY$.



Conversely,

If $OX = OY$, then

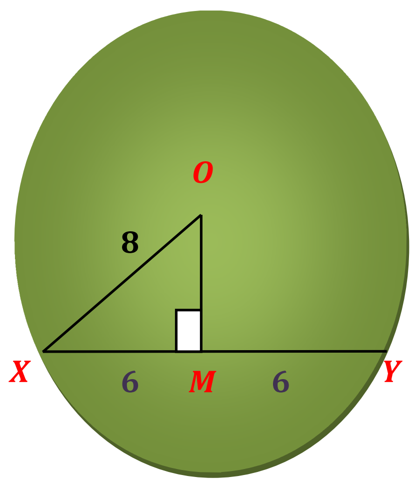
$PQ = RS$.

EXAMPLE8 :

A circle of radius 8 cm has centre O. A chord XY is 12 cm long. Calculate the distance from O to the midpoint of the chord.

SOLUTION:

Let M be the mid point of the chord XY.



$XM = \frac{1}{2} XY$

$= \frac{1}{2}(12)$

$= 6$ cm

$\angle OMX = 24^0$

If $XM = MY$, then

$OM \perp XY$.

Using Pythagoras' Theorem on OXM,

$OM^2 + 6^2 = 8^2$

$OM^2 = 8^2 - 6^2$

$= 28$

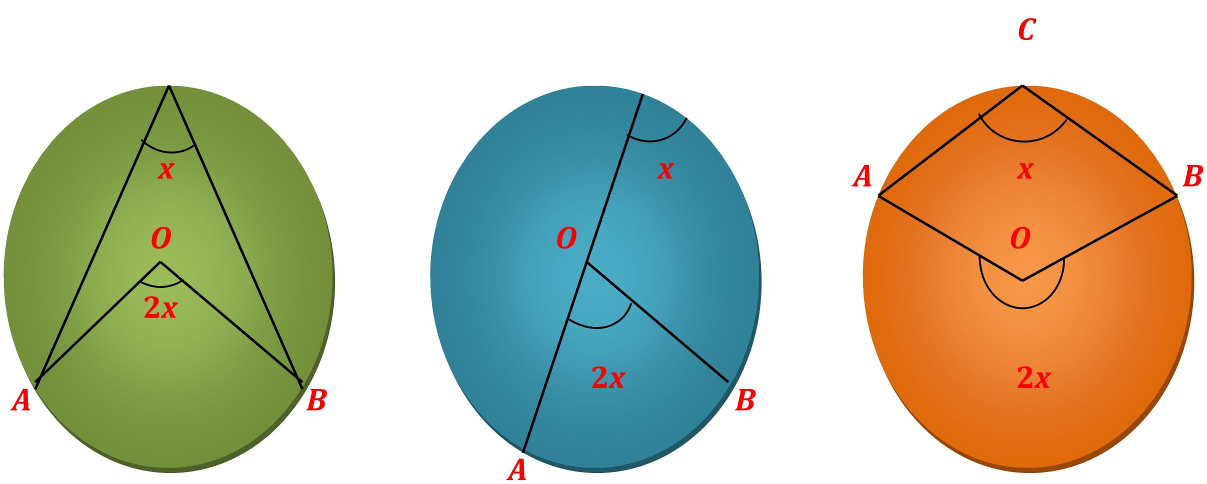
$OM = \sqrt{28}$

$\approx$ 5.29 cm   (correct to 3 sig. fig .)

$\therefore$ The required distance is 5.29 cm .

1.6 ANGLE PROPERTIES OF CIRCLE:

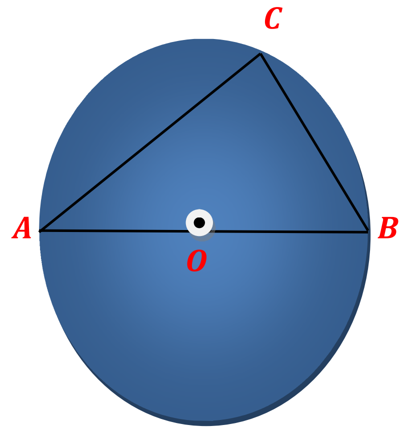
1. An angle at the centre of a circle is twice the angle at the circumference subtended by the same arc .



$\angle AOB = 2\angle ACB$ ($\angle$ At centre = 2 $\angle$ At circumference)

2. **The angle in a semicircle is a right angle .**

If *AB* is the diameter of the circle, centre *O* , then

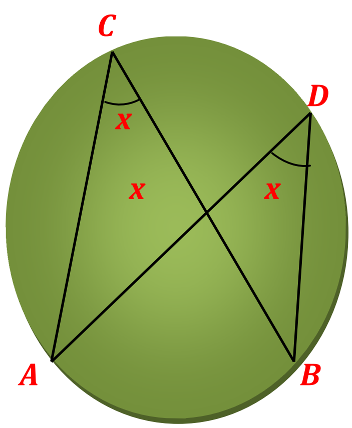


$\angle ACB = 90^0$

(rt. ∠ in semicircle)

2. **The angle in a semicircle is a right angle .**

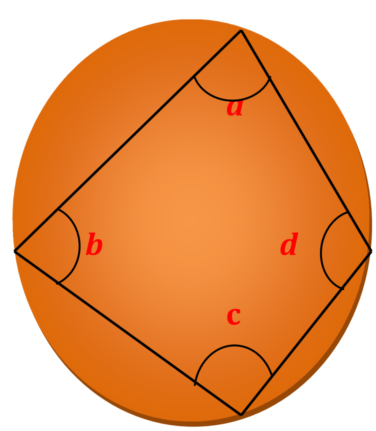
If *AB* is the diameter of the circle, centre *O* , then



$\angle ACB = 90^0$

(∠ s in the same segment)

4. **The angle in opposite segments of a circle are supplementary (i.e. the sum of the angles add up to 180⁰.)**



$a + c = 180^0$
$b + c = 180^0$

(∠ s in opp. Segments are supp .)

TIPS FOR STUDENTS:

1. A cycle quadrilateral is a quadrilateral drawn inside a circle so that all its 4 vertices lie on the circumference of the circle .

2. The opposite angles of a cyclic quadrilateral add up to 180^0 . (Supplementary angles)

EXAMPLE9 :

In the diagram, *O* is the center of the circle *PQRST* . *POR* is the diameter of the circle, $\angle ROS = 72^0$ and $\angle QPR = 25^0$.

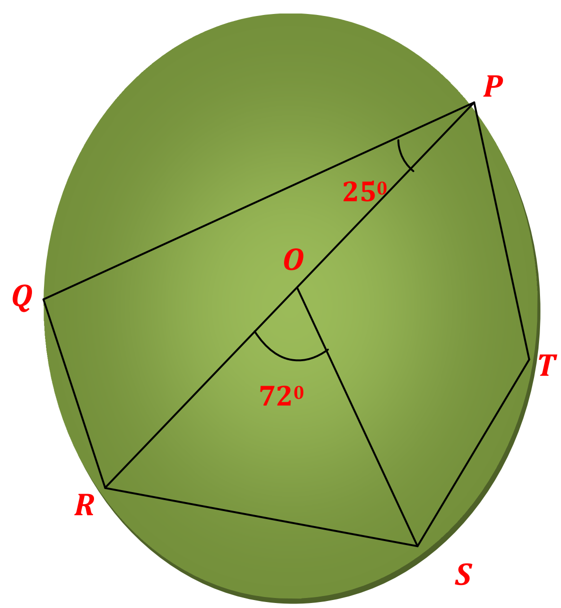
FIND

a) $\angle RPS$,

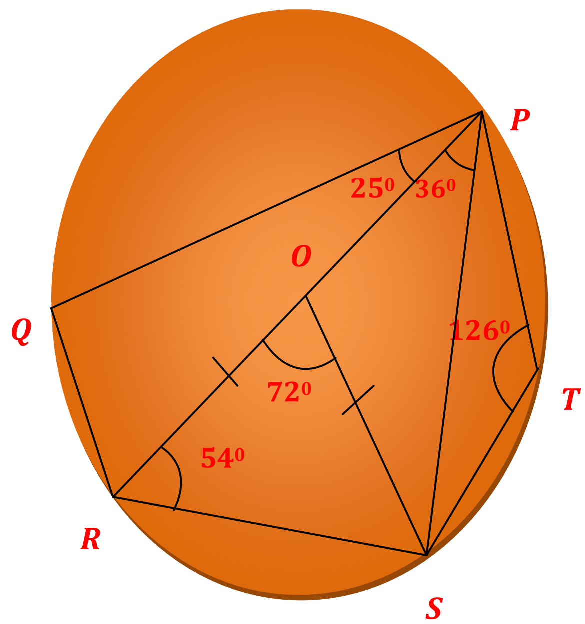
b) $\angle QES$,

c) $\angle PTS$,

d) $\angle PRQ$.



SOLUTION :



a) $\angle RPS$ = $72^0 \div 2$ ($\angle$ s at centre = 2 $\angle$ at circumference)

= 36^0

b) $\angle QES$ = $\frac{180^0 - 72^0}{2}$ (base $\angle$ s of isos . Δ)

= 54^0

c) $\angle PTS$ = 180^0 - $\angle PRS$ ($\angle$ s in opp. Segments are supp .)

= 180^0 - 54^0

= 126^0

d) $\angle PRQ$ = 90^0 (rt . $\angle$ s in Semicircle.)

= 180^0 - 90^0 - 25^0 ($\angle$ s sum of Δ)

$= \quad 65^0$

EXAMPLE10 :

The points *A*, *B*, *C* , *D* and E lie on the circle . $\angle AEB = 59^0$, $\angle ADE = 23^0$, $\angle BDC = 48^0$ **and *AB* is parallel to *DC* .**

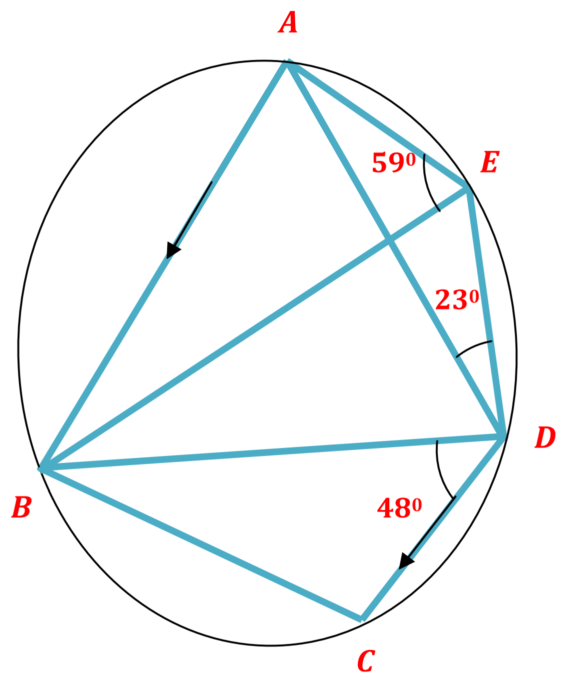
FIND

a) $\angle ABE$

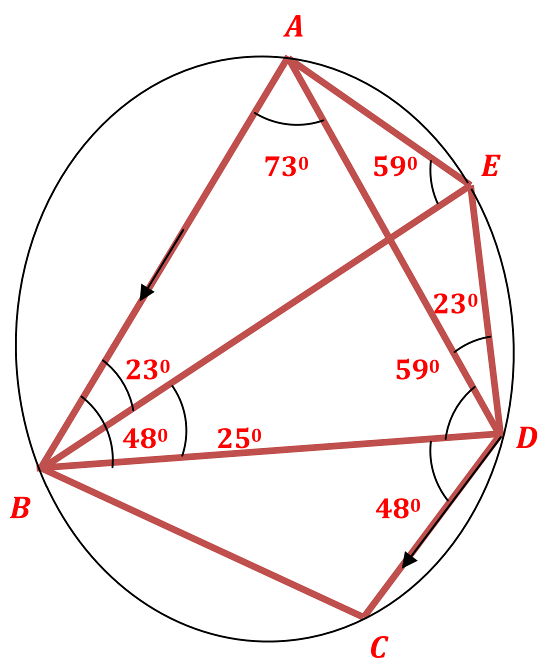
b) $\angle DBE$,

c) $\angle BAD$,

d) $\angle BCD$.



SOLUTION :



a) $\angle ABE$ = $\angle ADE$ ($\angle$ s in the same segment)

= 23^0

b) $\angle ABD$ = $\angle BDC$ (alt . $\angle$ s $AB // DC$)

= 48^0 - 23^0

= 25^0

c) $\angle ADB$ = $\angle AEB$ ($\angle$ s in the same segment)

= 59^0

$\angle PTS$ = 180^0 - $\angle ABD$ - $\angle ADB$ ($\angle$ s sum of Δ)

= 180^0 - 48^0 - 59^0

= 73^0

d) $\angle BCD$ = 90^0 - $\angle BAD$ ($\angle$ s in opp. Segments are supp .)

= 180^0 - 73^0

= 107^0

EXAMPLE11 :

In the diagram, *A*, *B*, *C* and *D* lie on the circle . *ADE* and *BCE* are straight lines .

BAC = 41^0, *ABD* = 48^0 and *DCE* = 75^0.

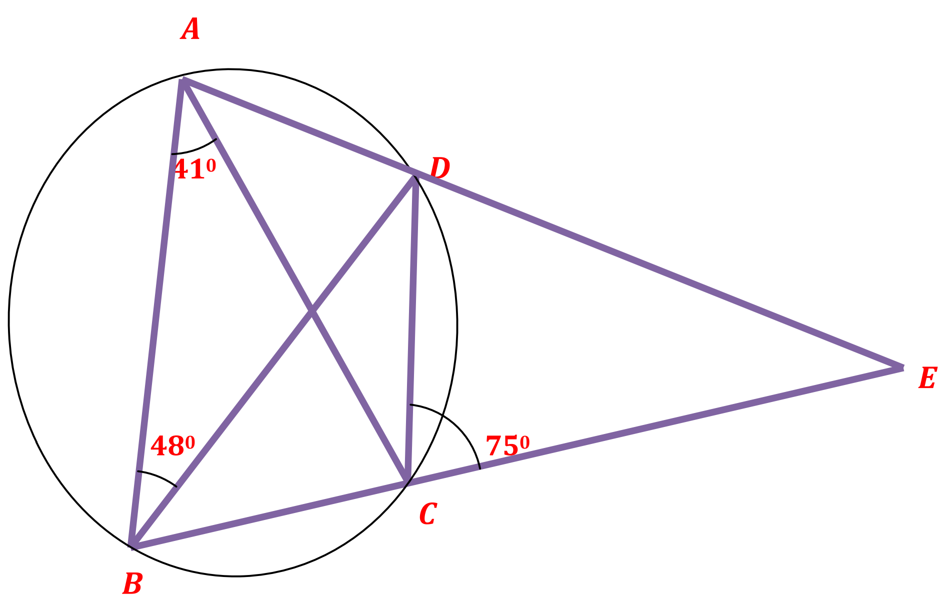
FIND

a) $\angle BDC$,

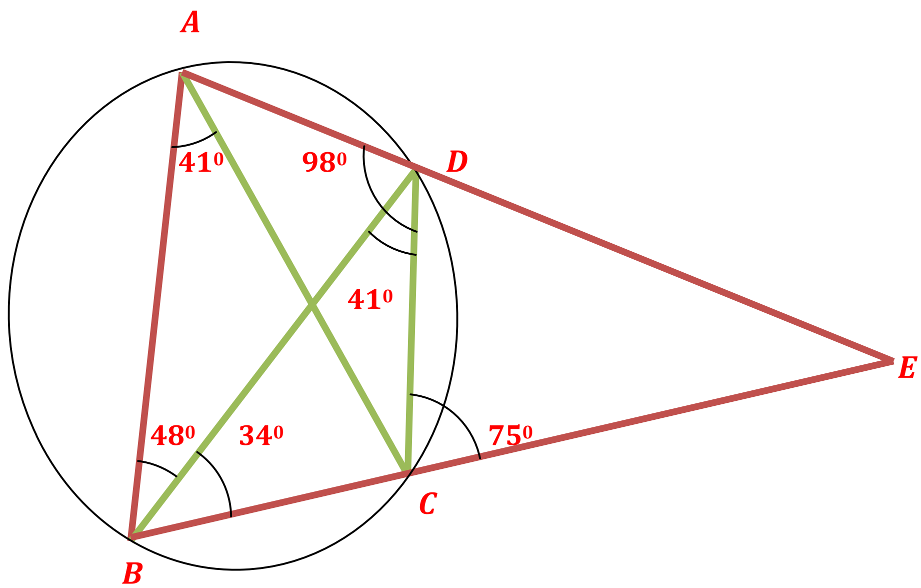
b) $\angle CBD$,

c) $\angle ADC$,

d) $\angle CED$.



SOLUTION:



a) $\angle BDC = \angle BAC$ ($\angle$ s in the same segment)

$= 41^0$

b) $\angle ABD + 41^0 = 75^0$ (ext. $\angle$ s of Δ)

$\angle CBD = 34^0$

c) $\angle ADC = 180^0 - \angle ABC$ ($\angle$ s in opp. Segments are supp .)

$= 180^0 - (48^0 + 34^0)$

$= 98^0$

d) $\angle CED + 75^0 = 98^0$ (ext . $\angle$ s of Δ)

$\angle CED = 23^0$

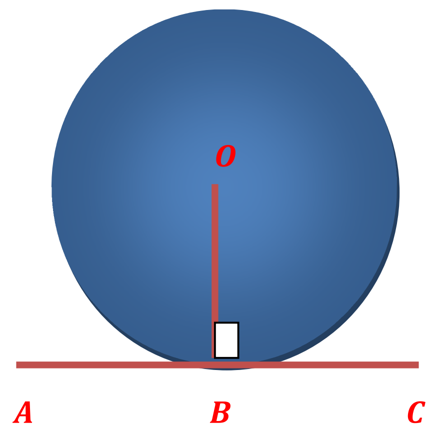
1.7 TANGENT THEOREMS:

A tangent to a circle is a line which touches the circle at only one point . A tangent perpendicular to the radius at the point of contact .

If *ABC* is a tangent to the circle at *B*, then

$\angle OBA = \angle OBC = 90^0$

(tan $\perp$ rad .)

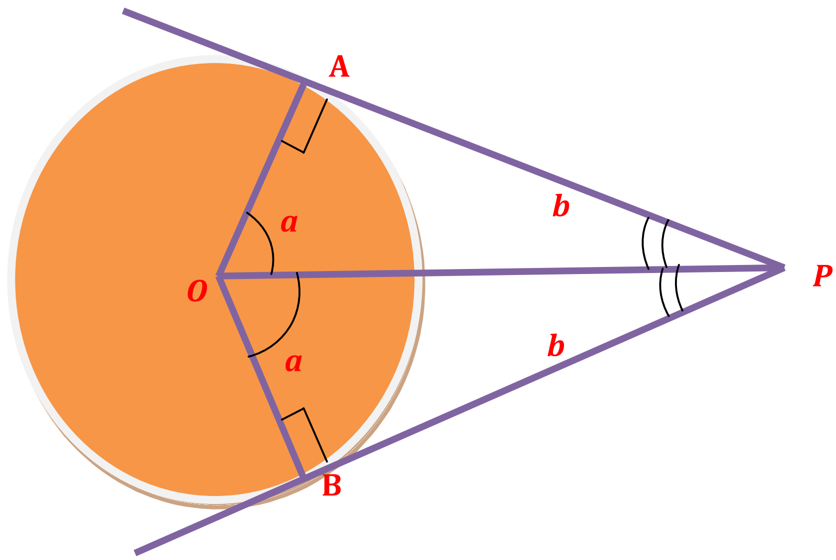


TANGENTS FROM AN EXTERNAL POINT

1. Tangents drawn from an external point to a circle are equal .

If *PA* and *PB* are tangents to the circle, then

$PA = PB$



2. The tangents subtend equal angles at the centre .

If *PA* and *PB* are tangents to the circle center *O*, then

$\angle POA = \angle POB$

3. The lie joining the external point to the centre of the circle bisects the abgles between the tangents .

If *PA* and *PB* are tangents to the circle, center *O*, then

$\angle APO = \angle BPO$

EXAMPLE11 :

The tangents *TA* and *TD* are drawn from a point *T* to the circle, center *O*. The diameter *DB* and the tangent *TA* when produced meet at *E*. Given that $\angle ADO = 26^0$.

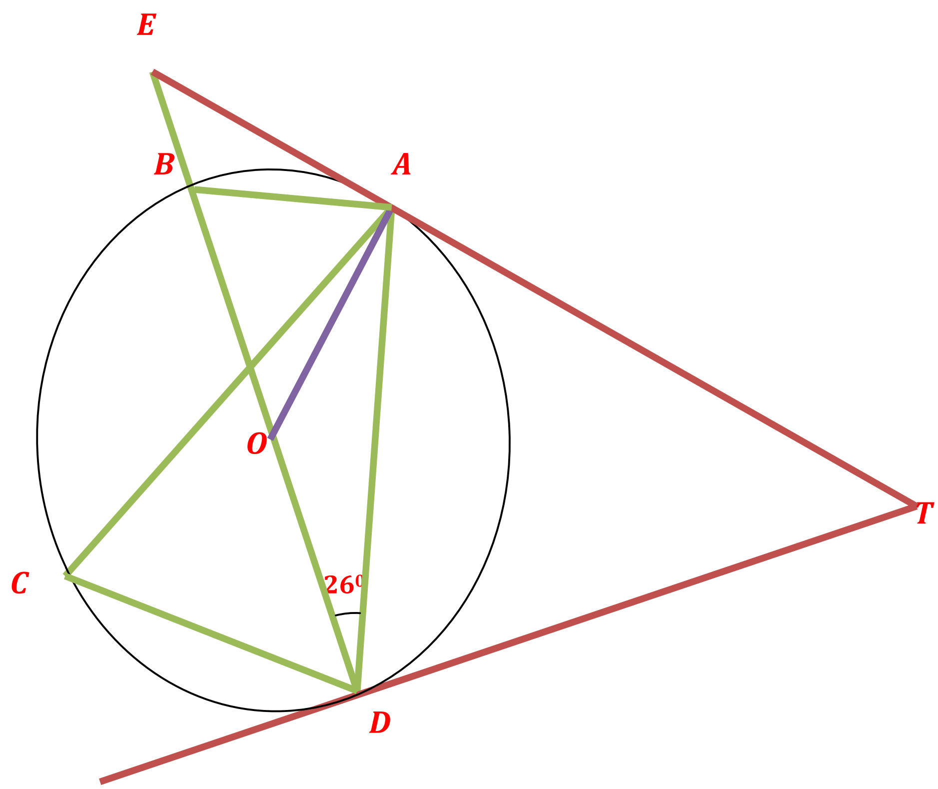
FIND

a) $\angle ADT$,

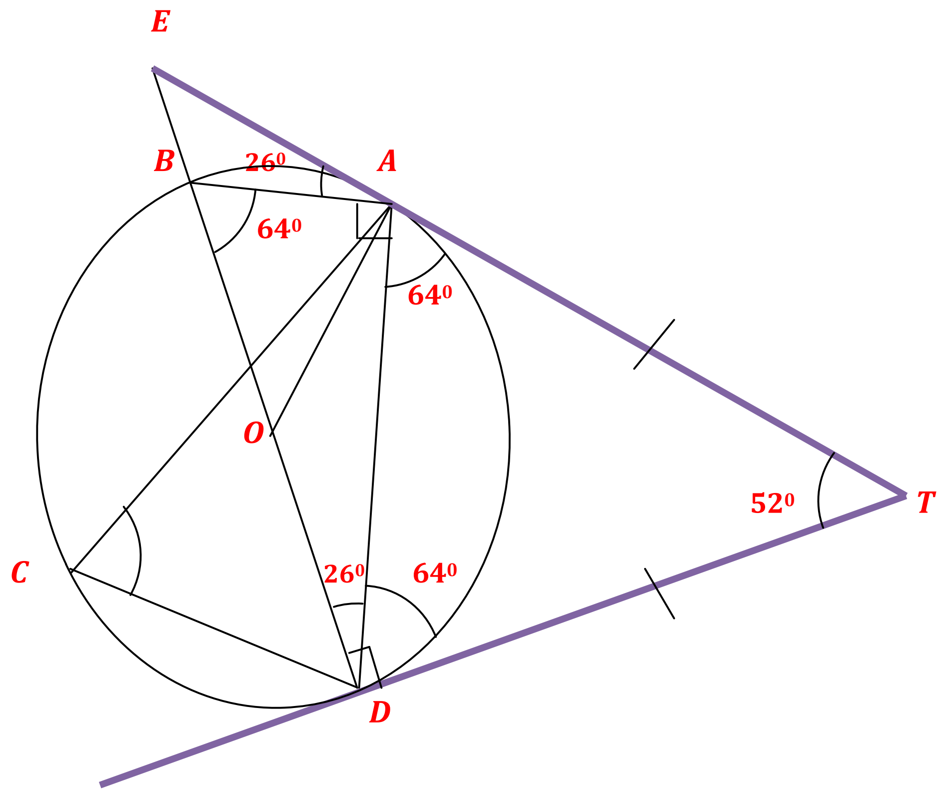
b) $\angle ATD$,

c) $\angle BAE$,

d) $\angle ACD$.



SOLUTION :



a) $\angle ODT = 90^0$ (tan $\perp$ rad .)

$\angle ODT = 90^0 - 26^0$

$= 64^0$

b) $TA = TA$ (tangents from an external point)

$\angle DAT = \angle ADT = 64^0$ (base $\angle$ s isos . Δ)

$\angle ADT = 180^0 - 64^0 - 64^0$ ($\angle$ s sum of Δ)

$= 52^0$

c) $\angle BAD = 90^0$ (rt. $\angle$ s in semicircle.)

$\angle BAE = 180^0 - 90^0 - 64^0$ **(adj . ∠ s on a str . line)**

$= 26^0$

d) $\angle ABD = 180^0 - 90^0 - 26^0$ **(∠ s of sum Δ)**

$= 64^0$

$\angle ACD = \angle ABD$ **(∠ s in the same segment)**

$= 64^0$

EXAMPLE12 :

In the diagram , *A*, *B*, *C* and *D* lie on the circle, center . *SCT* is a tangent to the circle at *C*, *AO* is parallel to *BC* and $\angle BCS = 42^0$ **.**

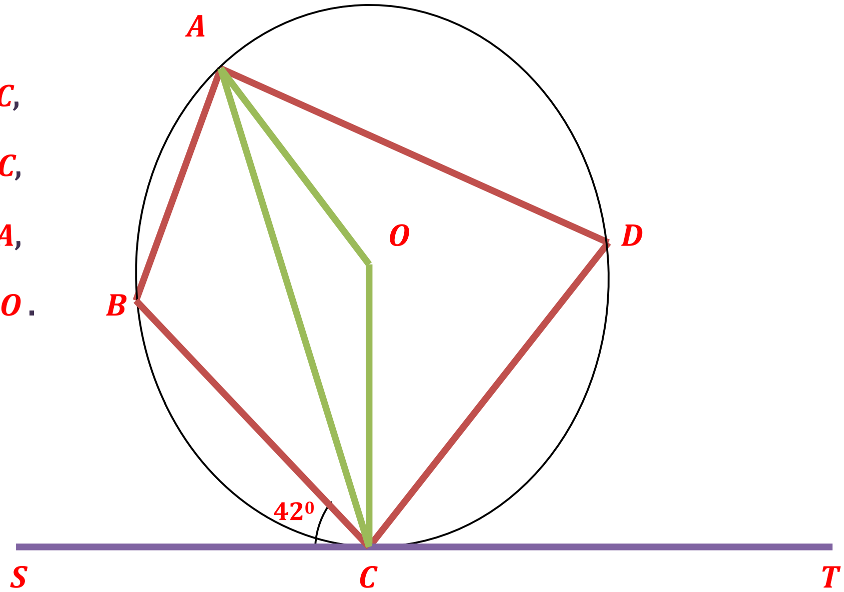
FIND

a) $\angle AOC$,

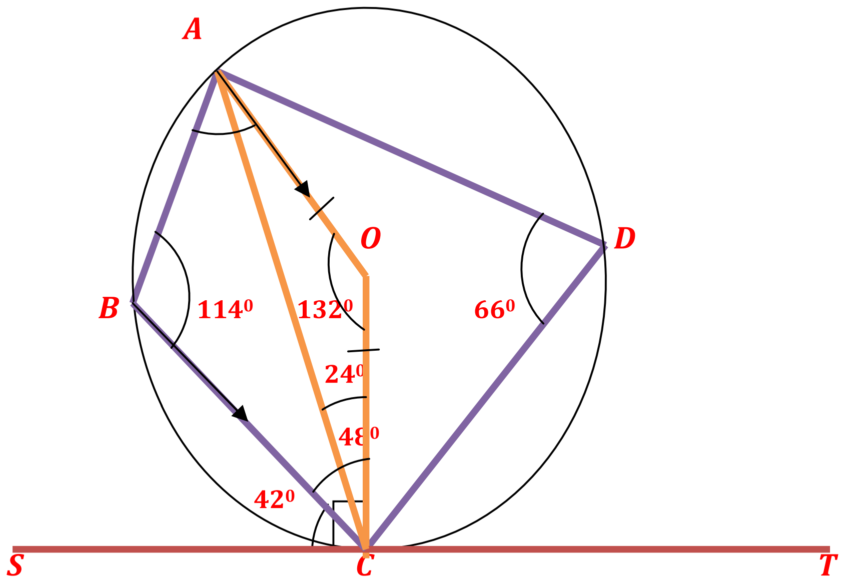
b) $\angle ADC$,

c) $\angle OCA$,

d) $\angle BAO$.



SOLUTION :



a) $\angle OCS = 90^0$ (tan ⊥ rad .)

$\angle OCB = 90^0 - 42^0$

$= 48^0$

$\angle AOC = 180^0 - 48^0$ (int . ∠ s AO // BC)

$= 132^0$

b) $\angle ADC = \frac{1}{2} \times \angle AOC$ (at center = 2 ∠ at circumference .)

$= \frac{1}{2} \times 132^0$

$= 66^0$

c) $\angle OCA = \frac{180^0 - 132^0}{2}$ (base $\angle$ s of isos . Δ)

$= 24^0$

d) $\angle ABC = 180^0 - 66^0$ ($\angle$ s in opp . segment are supp .)

$= 114^0$

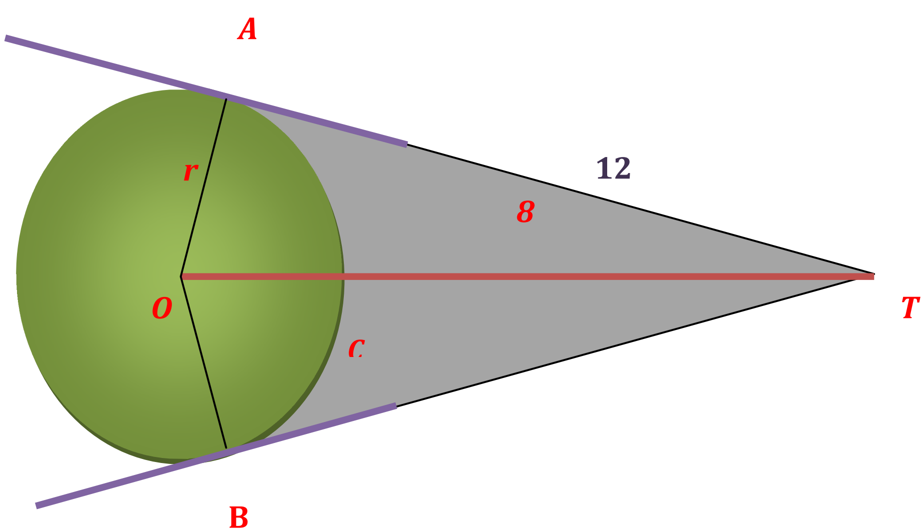
$\angle BAO = 180^0 - 114^0$ (int . $\angle$ s AO // BC)

$= 66^0$

EXAMPLE13 :

TA and *TB* are tangents to a circle, center *O* and radius *r* cm .

Give that $AT = 12$ cm and $CT = 8$ cm .



FIND

a) An expression for *OT* in term of *r* ,

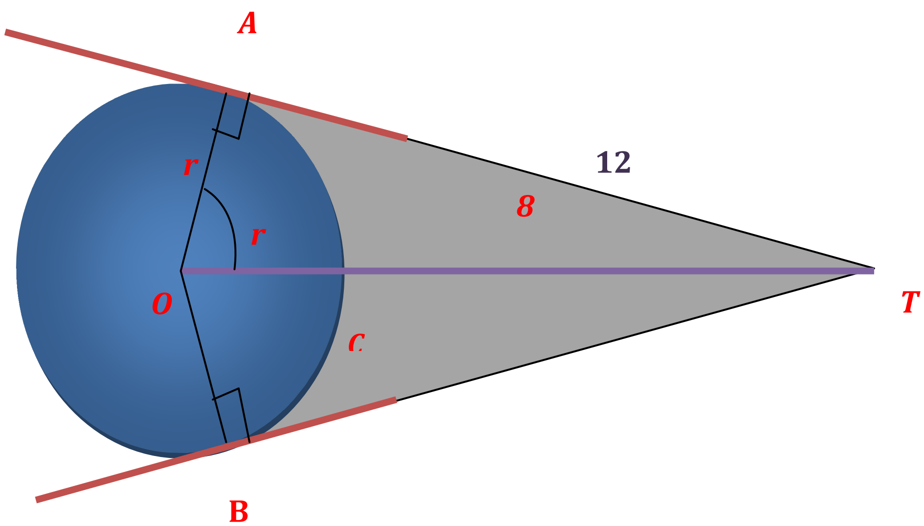
b) The value of r ,

c) The area of quadrilateral $OATB$,

d) $\angle AOB$ un radians ,

e) The area of the shaded region .

SOLUTION:



a) $OT = r$ cm (radius of circle)

$OT = (r + 8)$ cm

b) $\angle OAT = \angle OBT = 90^0$ (tan $\perp$ rad .)

Using Pythagoras' Theorem on $\Delta\ OAT$,

$$OT^2 = OA^2 + AT^2$$

$$(r + 8)^2 = r^2 + 12^2$$

$$r^2 + 16r + 64 = r^2 + 144$$

$$16r = 80$$

$$r = 5 \text{ cm}$$

c) Area of quadrilateral $OATB$,

$= 2 \times$ Area of $\Delta\, OAT$

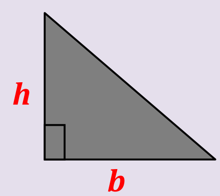
$= 2 \times \left(\frac{1}{2} \times 12 \times 5 \right)$

$= 60 \text{ cm}^2$

Area of Δ

$\frac{1}{2} \times$ Base $\times$ Height

$\frac{1}{2} \times b \times h$



d) $\tan \angle AOT = \frac{12}{5}$

$\angle AOT = \tan^{-1} \frac{12}{5}$

$= 1.176$ rad

$\angle AOB = 2 \times \angle AOT$

$= 2 \times 1.176$

$= 2.352$

$= 2.35$ rad (correct to 3 sig. fig.)

e) Area of shaded region

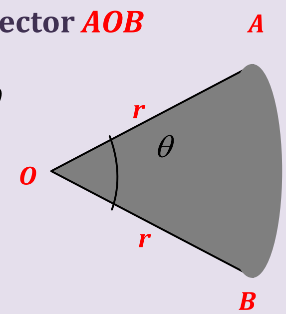
= Area of quadrilateral *OATB* – Area

$= 60 - \frac{1}{2} \times 5^2 \times 2.352$

$= 30.6 \text{ cm}^2$ (correct to 3 sig . fig .)

Area of sector *AOB*

$= \frac{1}{2} r^2 \theta$

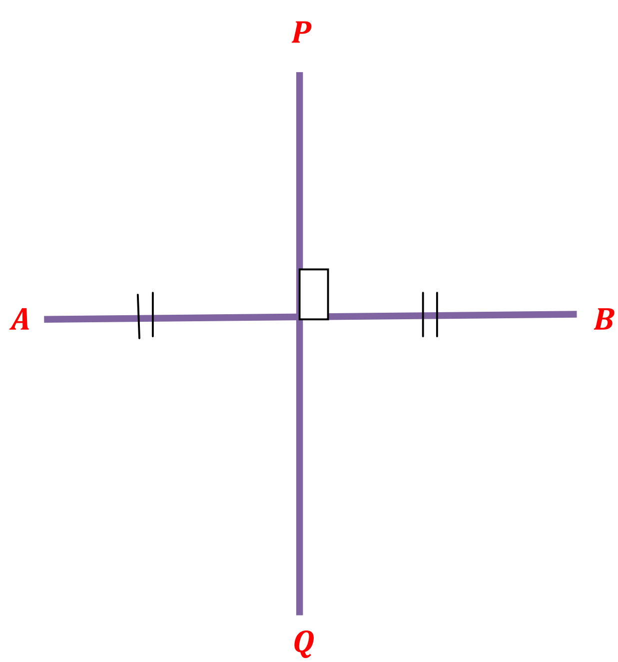


1.8 PERPENDICULAR BISECTS AND ANGLE BISECTORS:

PERPENDICULAR BISECTOR OF A LINE

1. The Perpendicular Bisector of a line segment forms a right angle with the line segment and divides the line segment into equal parts .

e.g.



PQ **is called** ***perpendicular bisector for line segment AB.***

PROPERTY OF PERPENDICULAR BISECTOR (⊥ Bisector)

Any point on the perpendicular bisector of a line segment is equidistant to the two end points of the line segment .

2. To construct the perpendicular bisector of the line *AB* :

a) Set your compasses to more than half the length of *AB* .

b) With A as center, mark arcs above and below *AB* . (Figure 1)

c) Repeat the process with *B* as the centre . (Figure 1)

d) Draw a straight line that joins *P* and *Q* (where the two sets of arcs intersect) .

(Figure 1) *PQ* is the perpendicular bisector of *AB* .

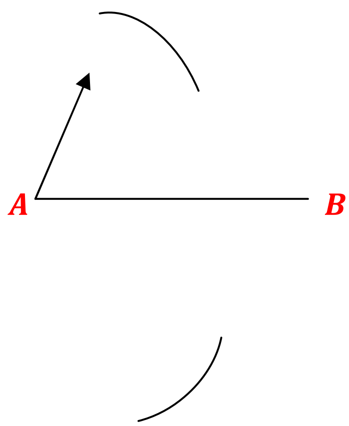


Figure 1

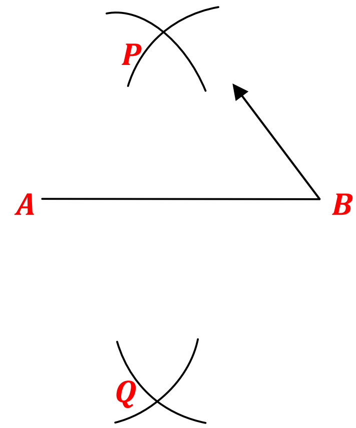


Figure 2

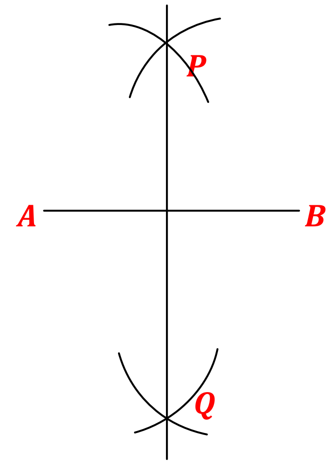
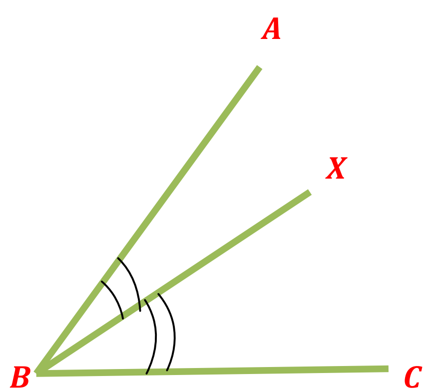


Figure 3

ANGLE BISECTOR:

1. An angle bisector is a ray that divides an angle into equal angles.

e.g.



If BX splits $\angle ABC$ into two angles such that $\angle ABC = \angle ABC$, then BX is the angle bisector of $\angle ABC$.

PROPERTY OF ANGLE BISECTOR (⊥ Bisector)

Any point on the angle bisector is equidistant from the two sides of the angle .

2. To construct the angle bisector of ∠ *ABC* :

a) Use a pair of compasses and with *SB* as centre, draw an arc to cut *AB* at *X* and *BC* at *Y* .

(Figure 1)

b) Using same radius, with *X* and *Y* as centers, draw two arcs that intersect at *Z* .

(Figure 2)

c) Draw a straight line from *B* through *Z* . (Figure 3)

***BZ* is the angle bisector of ∠ *ABC* .**

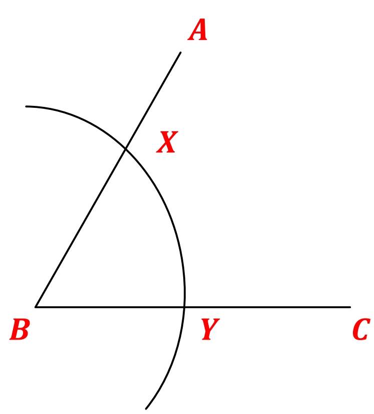


Figure 1

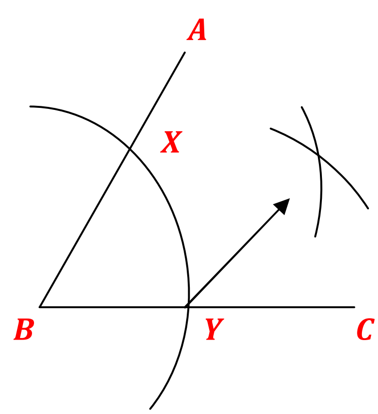


Figure 2

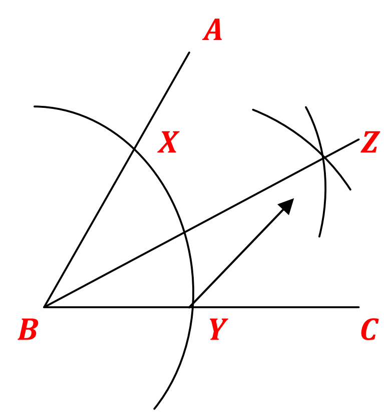


Figure 2

1.9 CONSTRUCTIONS OF SIMPLE GEOMETRIC FIGURES:

Draw a rough sketch and write down the given information before constructing the actual figure.

EXAMPLE14 :

Draw a triangle ABC in which $AB = 7.5$ cm, $\angle ABC = 110^0$ and $BC = 8$ cm .

a) Measure and write down the length of AC .

b) On the triangle, construct

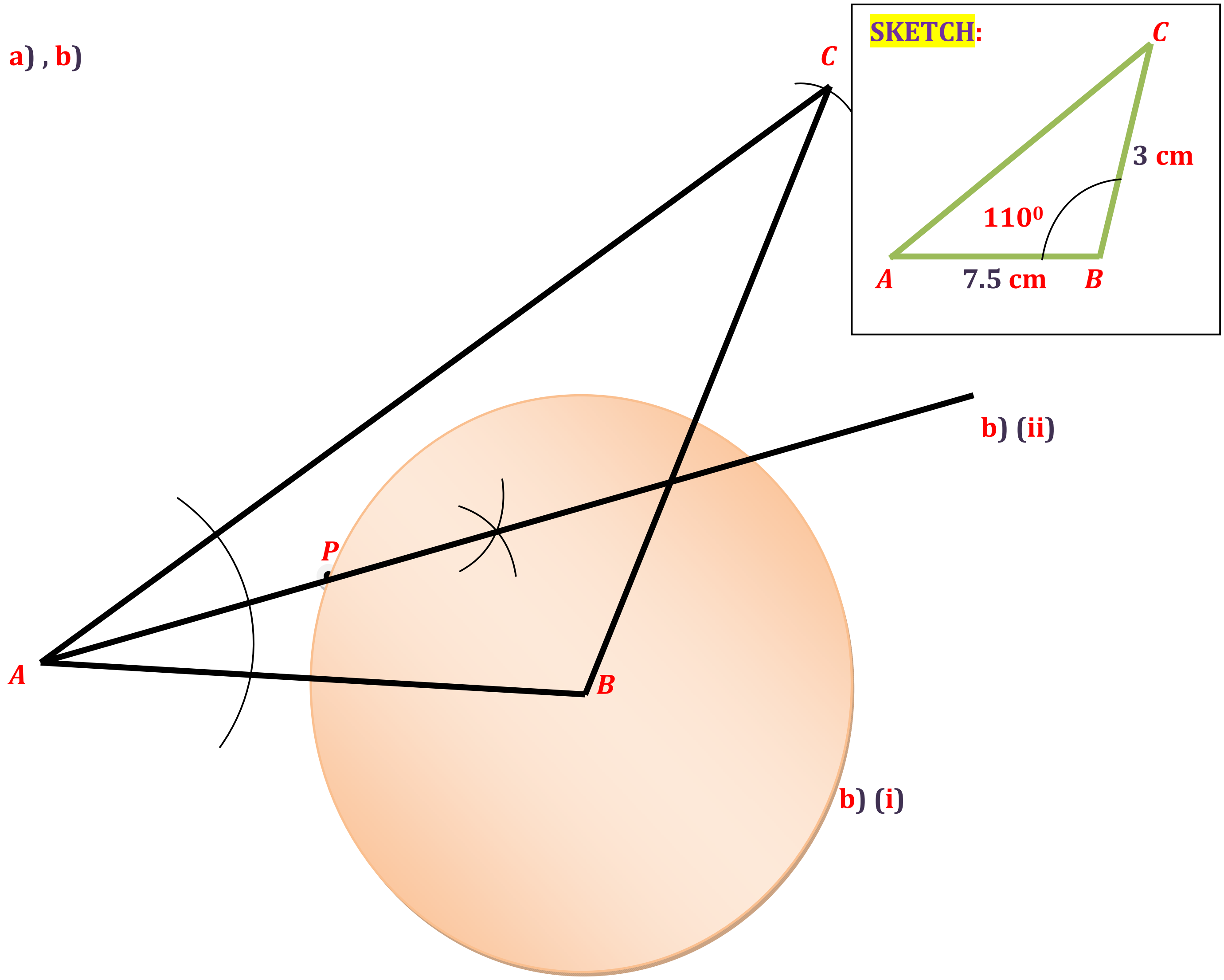
i) A circle of radius 3.8 cm with centre B ,

ii) The angle bisector of $\angle BAC$.

c) The angle bisector cuts the circle at the point P , given that P lies inside triangle ABC , Complete the sentence below.

The point P is 3.8 cm from the point________ and is equidistant from the lines_________ and_____________ .

SOLUTION :



a) By measurement, the length of ***AC*** is **12.7** cm .

b) The point ***P*** is **3.8** cm from the point ***B*** and is equidistant from the lines ***AB*** and ***AC*** .

EXAMPLE15 :

Construct the quadrilateral ***ABCD*** in which the base ***AB*** = **6** cm, ∠***ABC*** = **108⁰**, ***BC*** = **5** cm ,

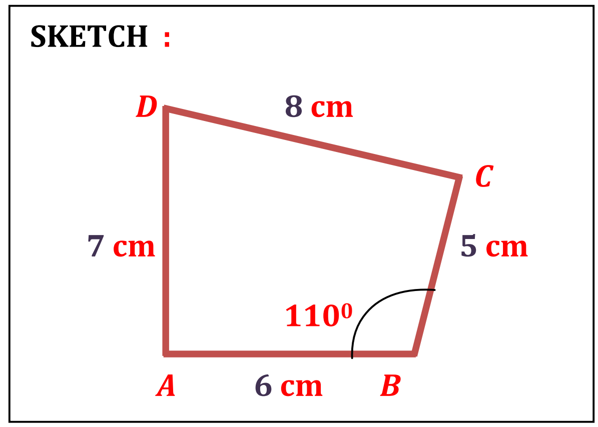
- **a)** Measure and write down the size of ∠***ADC*** ,
- **b)** On the quadrilateral, construct
  - **i)** The angle bisector of ∠***BCD*** ,

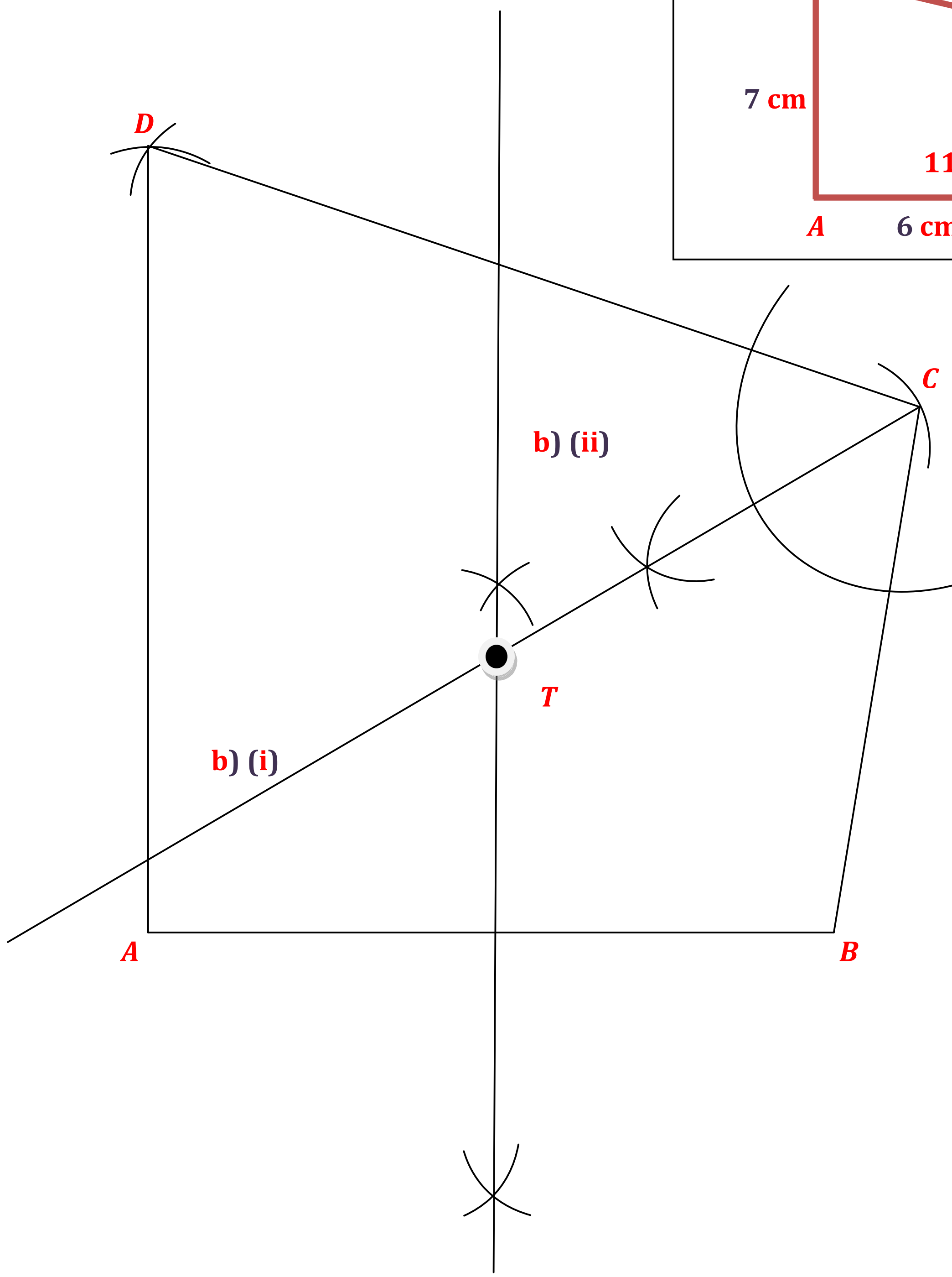
ii) The perpendicular bisector of AB .

c) These two lines intersect at the point T . Measure and write down the length of DT .

SOLUTION:

a) By the measurement, the size of $\angle ADC$ is 72.5^0





b) By the measurement, the length of DT is 5.6 cm.

PROPERTIES OF TRIANGLES

1.1 RELATION RETWEEN SIDES AND ANGLES OF A TRIANGLE:

1. A triangle consists of three sides and three angles called elements of the triangle.

   In any triangle ABC,

   A, B, C denotes the angles of the triangle at the vertices.

$$\therefore \quad A + B + C = 180^0$$

2. The sides of the triangle are denoted by a, b, c opposite to the angles A, B and C respectively .

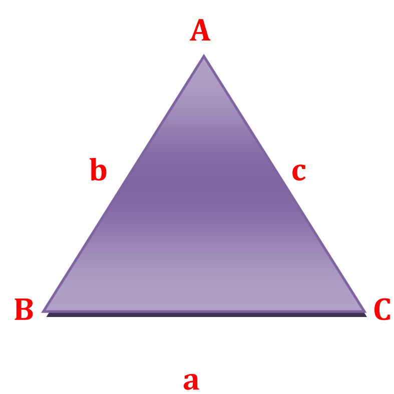


Fig (1)

3. $a + b + c = 2s$ = The perimeter of the triangle .

1.2 THE SINE RULE:

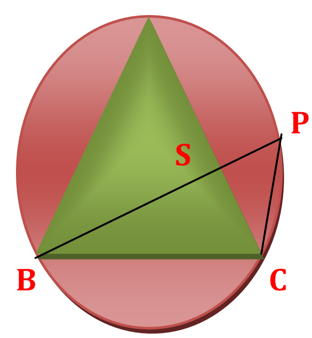
In a triangle ABC, prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Where, R is the circum radius of the triangle.

PROOF: Let S be the circumcentre of the triangle ABC. First prove that $\frac{a}{\sin A} = 2R$

CASE (I) : Let S be an acute angle. Let P be any point on the circle. Join BP, Which pass through S . Join CP, so that $\angle BCP = 90^0$. BAC = a = $\angle$BPC (angles in the same segment) .



FROM Δ BPC, $\sin B\widehat{P}C = \frac{BC}{BP}$

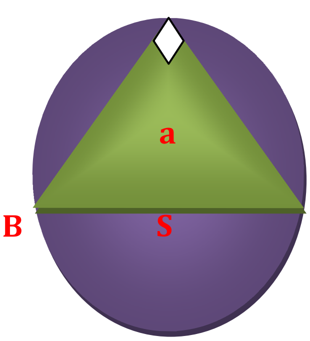
$\therefore \sin \widehat{A} = \frac{a}{2R}$, $\therefore \frac{a}{\sin A} = 2R$

Fig (2)

CASE (II) : Let A be right angle, ie., $\widehat{A} = 90^0$ (Fig 3), Then BC is the diameter.

BC = a = 2R

$\therefore \sin \widehat{A} = \frac{BC}{2R} = \frac{a}{2R}$



$\therefore \frac{a}{\sin A} = 2R$

Fig (3)

CASE (III) : Let A be an obtuse angle (Fig 4) . join BP, passing through S .

Join CP, so that $B\hat{C}P = 90^0$.

Now $B\hat{P}C = 180^0 - (B\hat{A}C) = 180^0 - A$

(Since ABPC is a cyclic quadrilateral)

From Δ BPC, sin $(B\hat{P}C)$

$= \frac{BC}{BP}$

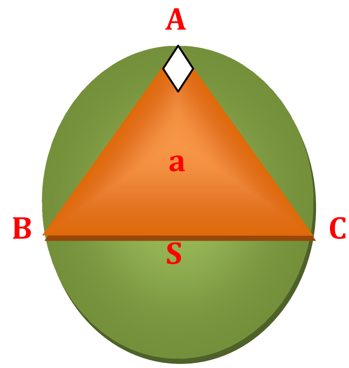


Fig (4)

i.e. $\sin(180^0 - A) = \frac{a}{2R}$

$\therefore \sin A = \frac{a}{2R} \qquad \therefore \frac{a}{\sin A} = 2R$

$\therefore \frac{a}{\sin A} = 2R$ is true for all values of A .

Similarly, we can prove, $\frac{b}{\sin A} = \frac{c}{\sin A} = 2R$,

Thus, $\boxed{\frac{a}{\sin A} = \frac{b}{\sin A} = \frac{c}{\sin A} = 2R.}$ or a = 2r sinA, b = 2r sinB, c = 2r sinC .

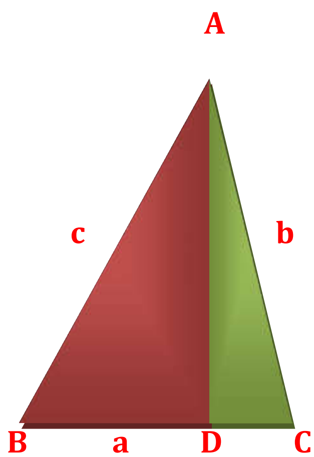
1.3 THE COSINE RULE:

In any triangle ABC, prove that

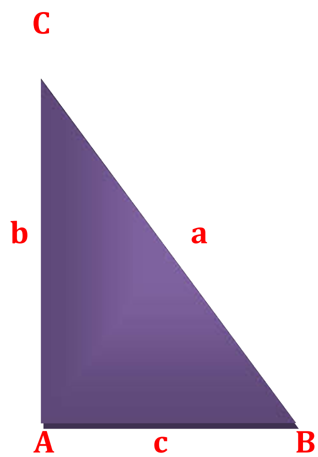
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

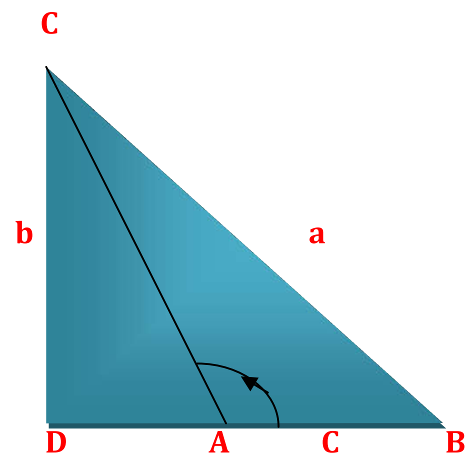
$$c^2 = a^2 + b^2 - 2ab \cos C$$



(Fig 5)



(Fig 6)



(Fig 7)

PROOF: Case (I) Let A be an acute angle (Fig 5)

Draw CD ⊥ AB . From Δ BDC

$$BC^2 = BD^2 + DC^2 = (AB - AD)^2 + DC^2$$

$$= AB^2 - 2AB\,.\,AD + AD^2 + DC^2$$

$$BC^2 = AB^2 - 2AB\,.\,AD + AC^2 \qquad (\text{Since } AD^2 + DC^2 = AC^2)$$

$$\therefore\ a^2 = c^2 - 2C\,.\,AD + b^2$$

But from Δ ADC ,

$$\text{Cos } A = \frac{AD}{AC}$$

$$\therefore\ AD = A \cos A = b \cos A$$

$$\therefore\ a^2 = c^2 - 2c.b.\cos A + b^2 \quad \text{Or}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Case (II) Let A be right angled, ie., $\hat{A} = 90^0$ (Fig 6)

$$\therefore\ BC^2 = BA^2 + AC^2 \quad \text{ie., } a^2 = b^2 + c^2$$

But $a^2 = b^2 + c^2 - 2bc \cos A$

$\Rightarrow$ $a^2 = b^2 + c^2 - 2bc \cos 90^0$

$a^2 = b^2 + c^2$, which is true for a right angled triangle .

Case (III) Let A be obtuse angle, ie., $A > 90^0$ (Fig 7) .

Draw CD $\perp$ BA produced.

From Δ BDC $BC^2 = BD^2 + DC^2 = (BA + AD)^2 + DC^2$

$\therefore BC^2 = BA^2 + 2BA.AD + AD^2 + DC^2$

$BC^2 = BA^2 + 2AB.AD + AC^2$ (Since, $AD^2 + DC^2 = AC^2$)

$a^2 = c^2 - 2.C.AD + b^2$

But from Δ ADC,

$\text{Cos}(D\hat{A}C) = \frac{AD}{AC}$, $(D\hat{A}C = 180^0 - A)$

$\therefore \text{Cos}(180^0 - A) = \frac{AD}{AC} = \frac{AD}{b}$

$-\cos A = \frac{AD}{b}$, $\therefore AD = b \cos A$

$\therefore a^2 = b^2 + 2bc(-b\cos A) + c^2$

$a^2 = b^2 + c^2 - 2bc\cos A$

Similarly, we can prove, $b^2 = c^2 + a^2 - 2ca\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Tips for Students:

The above formulae can be written as:

$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$, $\text{Cos } B = \frac{c^2 + a^2 - b^2}{2ac}$, $\text{Cos } C = \frac{a^2 + b^2 - a^2}{2ab}$,

These results are useful in finding the cosines of the angle when numerical values of the sides are given. Logarithmic computation is not applicable since the formulae involve sum and difference of terms. However, logarithmic method can be applied at the end of simplification to find angle

1.4 THE PROJECTION RULE:

In this rule, we show how, one side of a triangle can be expressed in terms of other two sides . It is called *projections* rule.

a = b cos C + c cos B ,

b = c cos A + a cos C , c = a cos B + b cos A .

PROOF: Let C be an acute angle

Draw AD ⊥ BC produced .

In Fig (i) BC = BD + DC

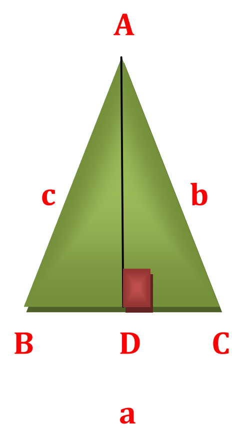
[NOTE : BD is called projection of AB on BC and DC is the projection of AC]

a = BD + DC …… (1)

From Δ BDA , cos B = $\frac{BD}{AB}$ ∴ BD = AB cos B = c cos B

From Δ CDA , cos C = $\frac{CD}{AC}$ ∴ CD = AC cos C = b cos C

From (1) c cos B + b cos C = b cos C + c cos B



(Fig 8)

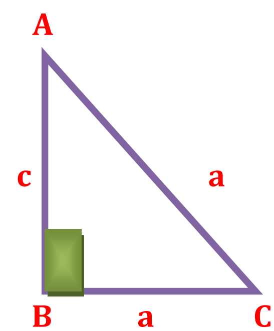
CaseII: When C is a right angle, ie., $\hat{C} = 90^0$ (Fig 9) .

$\cos B = \frac{BC}{AB} = \frac{a}{c}$, $\therefore$ $a = c \cos B$ ......(2)

Since $\hat{C} = 90^0$,

$\cos B = \cos 90^0 = 0$,

We get, $a = b \cos 90^0 + c \cos B \Rightarrow a = c \cos B$ ......(3)



(Fig 9)

Case III : When c is obtuse angle (Fig (iii))

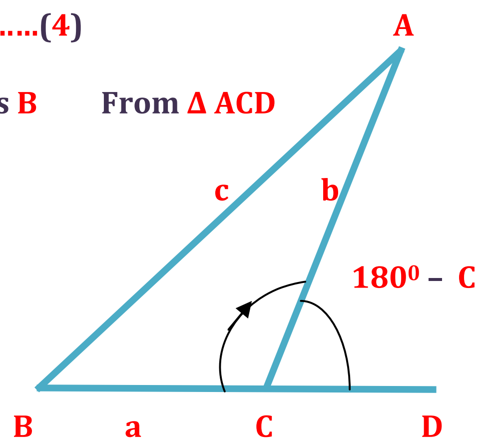
From Δ ABD, $BC = BD - CD$ ......(4)

From Δ ABD , $\cos B = \frac{BD}{AB} = \frac{BD}{c}$, $\therefore$ $BD = c \cos B$ From Δ ACD

$\text{Cos} (180^0 - C) = \frac{CD}{AC} = \frac{CD}{b}$

$\therefore \cos A = \frac{CD}{b}$, $\therefore$ $CD = - b \cos C$

From (4) $BC = c \cos B - (- b \cos C)$



(Fig 10)

ie., $a = c \cos B + b \cos C = b \cos C + c \cos B$

Similarly, $b = c \cos A + a \cos C$, $c = a \cos B + b \cos A$.

1.5 THE LAW OF TANGENTS:

In any Δ ABC , Prove that :

1. $\dfrac{a-b}{a+b} = \dfrac{\tan\left(\dfrac{A-B}{2}\right)}{\tan\left(\dfrac{A+B}{2}\right)}$,

2. $\dfrac{b-c}{b+c} = \dfrac{\tan\left(\dfrac{B-C}{2}\right)}{\tan\left(\dfrac{B+C}{2}\right)}$,

3. $\dfrac{c-a}{c+a} = \dfrac{\tan\left(\dfrac{C-A}{2}\right)}{\tan\left(\dfrac{C+A}{2}\right)}$

PROOF: **Using sine rule,**

$$\frac{a-b}{a+b} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B} = \frac{\sin A - \sin B}{\sin A + \sin B}$$

$$= \frac{2\cos\left(\dfrac{A+B}{2}\right).2\sin\left(\dfrac{A-B}{2}\right)}{2\cos\left(\dfrac{A+B}{2}\right).2\cos\left(\dfrac{A-B}{2}\right)}, = \cos\left(\frac{A+B}{2}\right).\tan\left(\frac{A-B}{2}\right)$$

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} \qquad \left(u\sin g\cot\theta = \frac{1}{\tan\theta}\right)$$

Similarly, other two results can be proved by changing sides and angles in cycle order.

1.6 EXPRESSIONS FOR HALF ANGLES IN TERMS OF a, b , c :

In any triangle ABC , prove that

1. $\sin\frac{A}{2} = \sqrt{\frac{(S-b)(S-C)}{bc}}$,

2. $\cos\frac{A}{2} = \sqrt{\frac{S(S-a)}{bc}}$,

3. $\tan\frac{A}{2} = \sqrt{\frac{(S-b)(S-c)}{S(S-a)}}$

PROOF: (1) We know that $2\sin^2 A = 1 - \cos A$

$$2\sin^2\frac{A}{2} = 1 - \frac{b^2+c^2-a^2}{2bc} \qquad \text{(Using cosine rule for A)}$$

$$2\sin^2\frac{A}{2} = \frac{2bc - b^2 + c^2 - a^2}{2bc}$$

$$= \frac{a^2 - (b^2 + c^2 - 2bc)}{2bc} = \frac{a^2 - (b-c)^2}{2bc}$$

$$= \frac{[a-(b-c)][a+b-c]}{2bc} = \frac{[a-b+c][a+b-c]}{2bc}$$

$$= \frac{(2S-2b)(2S-2c)}{2bc}$$

Since a + b + c = 2s

a + b = 2s - c

a + c = 2s - c

$$2\sin^2\frac{A}{2} = \frac{2(s-b)2(s-c)}{2bc}$$

(Divide by 2)

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{(S-b)(S-c)}{bc}}$$

If A is acute, then sin $\frac{A}{2}$ is always positive.

$$\therefore \sin\left(\frac{A}{2}\right) = \sqrt{\frac{(S-b)(S-c)}{bc}}$$

2\. $2\sin^2\frac{A}{2} = 1 + \cos A = 1 + \frac{b^2+c^2-a^2}{2bc}$ (Using cosine rule for A)

$2\sin^2\frac{A}{2} = \frac{2bc+b^2+c^2-a^2}{2bc} = \frac{(b+c)^2-a^2}{2bc}$

$2\sin^2\frac{A}{2} = \frac{(b+c+a)(b+c-a)}{2bc} = \frac{2s(2s-2a)}{2bc}$ (Using a + b + c = 2s, a + b = 2s – c)

Dividing by 2, we get

$\cos^2\frac{A}{2} = \frac{s(s-a)}{bc} = \pm\sqrt{\frac{S(S-a)}{bc}}$

Since $\frac{A}{2}$ is acute, $\cos\frac{A}{2}$ is always positive and therefore,

$\cos\frac{A}{2} = \sqrt{\frac{S(S-a)}{bc}}$

3\. $\tan\frac{A}{2} = \frac{\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}$

$= \frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\frac{s(s-a)}{bc}} = \frac{\sqrt{(s-b)(s-c)}}{s(s-a)}$

Similarly, we can show that

$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ac}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ac}}$, $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

WORKED EXAMPLES

1. ***If a = 3, b = 4, c = 5, in a triangle ABC , find the value of***

a.) $\sin\left(\frac{C}{2}\right)$ ***b.) sin 4C + cos 4C***

SOLUTION:

Since, $c^2 = b^2 + a^2$ is satisfied by the given sides, they form right angled triangle.

$5^2 = 4^2 + 3^2$

$\therefore \angle C = 90^0$, $\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{90^0}{2}\right)$

$= \sin 45^0 = \frac{1}{\sqrt{2}}$

And, $\sin 4C + \cos 4C = \sin(4 \times 90^0) + \cos(4 \times 90^0)$

$= \sin 360^0 + \cos 360^0$

$= 0 + 1 = 1$

2. *Prove that a sin (B – C) + b sin (C – A) + C sin (A – B) = 0 .*

SOLUTION:

Now, a sin(B – C) = 2R sin A . sin(B – C) (Since, a = 2R sin A)

= 2R sin A .sin(B – C)

(Since A + B + C = 180^0, B + C = 180^0 – A

sin(B + C) = sin A)

= 2R sin (B + C) sin (B – C) = 2R sin [$\sin^2$ B – $\sin^2$ C]

Similarly, b sin (C – A) = 2R [$\sin^2$ C – $\sin^2$ A]

C sin (A – B) = 2R [$\sin^2$ A – $\sin^2$ B]

∴ L. H. S. = a sin (B – C) + b sin (C – A) + C sin (A – B)

= 2R [$\sin^2$ B – $\sin^2$ C] + 2R [$\sin^2$ C – $\sin^2$ A] + 2R [$\sin^2$ A – $\sin^2$ B]

= 2R [$\sin^2$ B – $\sin^2$ C + $\sin^2$ C – $\sin^2$ A + $\sin^2$ A – $\sin^2$ B]

3. *Prove that, in a ΔABC ,* $\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2-c^2}{a^2}$

SOLUTION:

L. H. S. $= \dfrac{\sin(B-C)}{\sin(B+C)} \times \dfrac{\sin(B+C)}{\sin(B+C)}$

$$= \frac{\sin^2 B - \sin^2 C}{\sin(B+c)}$$

(**Since sin (B + C) sin (B - C) = $\sin^2$ B – $\sin^2$ C**)

$$= \frac{\sin^2 B - \sin^2 C}{\sin^2 A}$$

(**sin(A + B) = sin C in ΔABC**)

$$= \frac{\frac{b^2}{4R^2} - \frac{C^2}{4R^2}}{\frac{a^2}{4R^2}}$$

(using sine rule)

$$= \frac{\frac{b^2 - c^2}{4R^2}}{\frac{a^2}{4R^2}}$$

$$= \frac{b^2 - c^2}{a} = \textbf{R. H. S.}$$

4. *Prove that a(b cos C – c cos B) = b^2 - c^2*

SOLUTION:

L. H. S. = a(b cos C – c cos B) = ab cos C – ac cos B

$= ab \dfrac{a^2+b^2-a^2}{2bc}$ - $ac \dfrac{c^2+a^2-b^2}{2bc}$ (Using cosine rule)

$= \dfrac{a^2+b^2-a^2}{2bc} - \dfrac{c^2+a^2-b^2}{2bc}$

$= \dfrac{2b^2-2c^2}{2}$

= $b^2 - c^2$ = R. H. S.

5. ***Prove that*** $\dfrac{b^2-c^2}{a^2}\sin 2A + \dfrac{c^2-a^2}{b^2}\sin 2B + \dfrac{a^2-b^2}{c^2}\sin 2C = 0$

SOLUTION:

Now $\dfrac{b^2-c^2}{a^2}\sin 2A = \dfrac{b^2-c^2}{a^2} \times 2\sin A\cos A$ (Since, sin 2A = 2 sin A cos A)

$= \dfrac{b^2-c^2}{a^2} \times \dfrac{a}{2R} \times \dfrac{b^2+c^2-a^2}{2bc}$ (Using sine rule and cosine rule)

$= \dfrac{(b^2-c^2)(b^2+c^2-a^2)}{4R.abc}$

Similarly, $\dfrac{c^2-a^2}{b^2}\sin 2B = \dfrac{(c^2-a^2)(c^2+a^2-b^2)}{4R.abc}$

$\dfrac{a^2-b^2}{c^2}\sin 2C = \dfrac{(a^2-b^2)(a^2+b^2-c^2)}{4R.abc}$

L. H. S = $\frac{(b^2-c^2)(b^2 \ +c^2-a^2)}{4R.abc}$ + $\frac{(c^2-a^2)(c^2 \ +a^2-b^2)}{4R.abc}$ + $\frac{(a^2-b^2)(a^2 \ +b^2-c^2)}{4R.abc}$

= $\frac{1}{4R.abc}$ [$b^4 - c^4 - (b^2 - c^2)a^2 + c^4 - a^4 - (c^2 - a^2)b^2 + a^4 - b^4 - (a^2 - b^2)c^2$]

= $\frac{1}{4R.abc}$ [0] = **R. H. S**

6. ***Find the greatest side of the triangle, whose sides are $x^2 + x + 1$, $2x + 1$, $x^2 - 1$.***

SOLUTION:

Let $a = x^2 + x + 1$, $b = 2x + 1$, $c = x^2 - 1$

Then, a is the greatest side. Therefore $\widehat{A}$ is the greatest angle.

$$\therefore \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{(2x+1)^2(x^2-1)^2-(x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2+4x+1+x^4-2x^2+1-x^4-x^2-1-2x^3-2x-2x^2}{2(2x^3+x^2-2x-1)}$$

$$\cos A = \frac{-(2x^3+x^2-2x-1)}{2(2x^3+x^2-2x-1)}$$

$$= -\frac{1}{2} = -\cos 60^0 = \cos(180^0 - 60^0)$$

$$= -\frac{1}{2} = \cos 120^0 \quad \therefore A = 120^0$$

Therefore, the greatest angle is 120^0

7. *If sin 2A + sin 2B = sin 2C in a Δ ABC, Prove that either $\hat{A} = 90^0$ or $\hat{B} = 90^0$.*

SOLUTION:

sin 2A + sin 2B = sin 2C

$2 \sin \frac{(2A+2B)}{2} . \cos \frac{(2A-2B)}{2} = \sin 2C$

$\left(\text{Using } \sin C + \sin D = 2 \sin \frac{C+D}{2} . \cos \frac{C-D}{2} \right)$

2 sin (A + B) . cos (A – B) = 2sin C cos C

(∵ sin (A + B) = Sin(180^0 – C) = sin C)

2sin C cos (A – B) = 2sin C cos C

Dividing by 2 sin C both sides , we get ,

Cos(A – B) = cos C

Also, Cos(A – B) = cos -C [Since, cos(-C) = cos C]

∴ A – B = ± C

When A – B = C , A = B + C

But, A + B + C = 180^0, gives

A - B = 180^0, i.e., 2A = 180^0 , ∴ $\hat{A} = 90^0$

When A – B = - C , B = A + C

A + B + C = 180^0, gives

B + B = 180^0, i.e., 2B = 180^0 , ∴ $\hat{B} = 90^0$

Therefore, triangle is right angled triangle.

8. *Prove that* $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$

SOLUTION:

L. H. S $= \frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$ (**Using cos 2A = 1 – 2 sin² A**)

$= \frac{1-2\sin^2 A}{a^2} - \frac{1-2\sin^2 B}{b^2}$

$= \frac{1}{a^2} - \frac{2\sin^2 A}{a^2} - \frac{1}{b^2} + \frac{2\sin^2 B}{b^2}$

$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(\frac{\sin A}{a}\right)^2 + 2\left(\frac{\sin B}{a}\right)^2$

$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(\frac{1}{2R}\right)^2 + 2\left(\frac{1}{2R}\right)^2$

(**Since,** $\frac{\sin A}{a} = \frac{1}{2R}$, $\frac{\sin B}{b} = \frac{1}{2R}$)

$= \frac{1}{a^2} - \frac{1}{b^2}$ = **R. H. S**

SUMMARY AND KEY POINTS

1.)a) Acute angle: Angles less than 90^0.

b) Obtuse angle: **Angles greater than 90^0 but less than 180^0.**

c) Straight angle: **Angles equal to 180^0.**

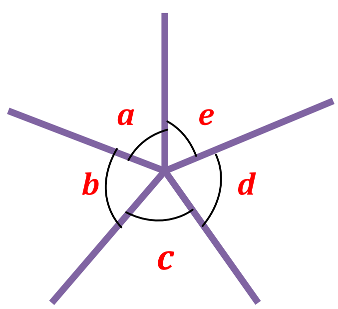
d) Reflex angle: Angles greater than 180^0 but less than 360^0.

2.) COMPLEMENTARY ANGLES: Two angles are complementary if their sum add up to 90^0.

3.) SUPPLEMENTARY ANGLES: Two angles are supplementary if their sum add up to 180^0 .

4.) The sum of **ADJACENT ANGLES** on a straight line is equal to 180^0.

5.) Angles at a point add up to 360^0.



$$a + b + c + d + e = 360^0$$

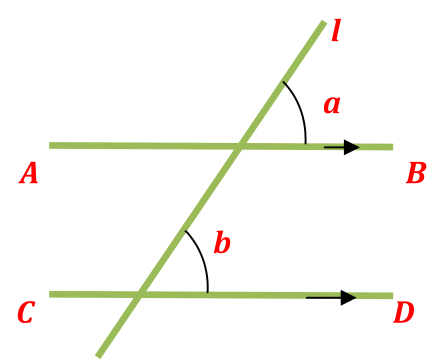
6.) Vertically opposite angles are equal.

7.) Angles formed by parallel lines cut by a transversal Line , *l* .

a) **CORRESPONDING ANGLES** are equal.

$a = b$

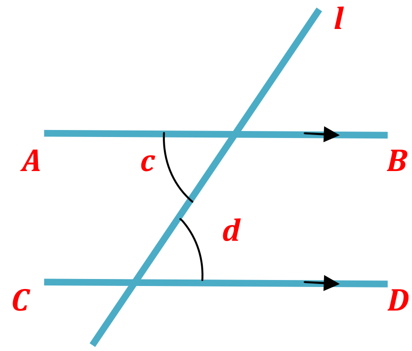
(corr. ∠s, *AB* // *CD*)



b) **ALTERNATE ANGLES are equal.**

$c = d$

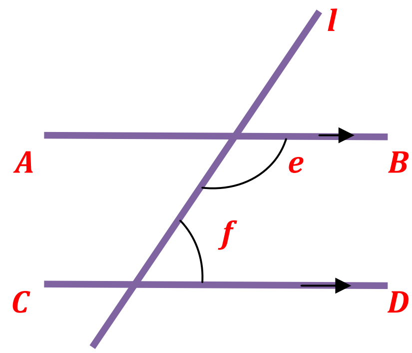
(alt. ∠s, *AB* // *CD*)



c) **INTERIOR ANGLES are supplementary.**

$e = f = 180^0$

(int. ∠s, *AB* // *CD*)

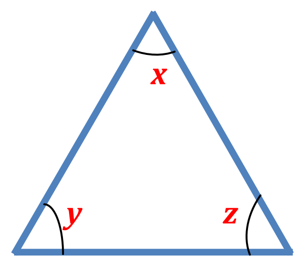


8.) ANGLE PROPERTIES OF TRIANGLES:

a.) The sum of the 3 angles of a triangle is equal to 180^0.

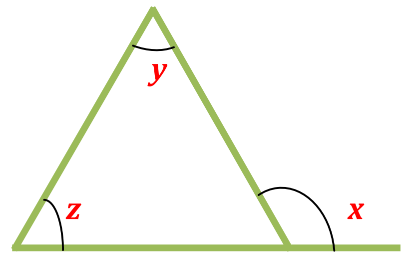
$x + y + z = 180^0$

(∠ sum of Δ)



b.) The exterior angle of a triangle is equal to the sum of the interior opposite angles.

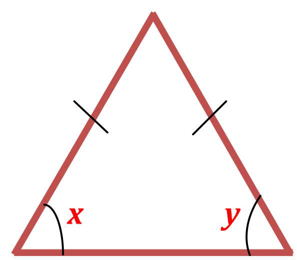
$x = y + z$



(ext . ∠ of Δ)

c.)An Isosceles Triangle has 2 equal angles opposite the 2 equal sides.

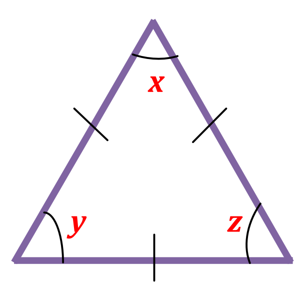
$x = y$



(base ∠ s of isos . Δ)

d.)An Equilateral Triangle has 3 equal sides and 3 equal angles, each equal to 60^0 .

$x + y + z = 60^0$

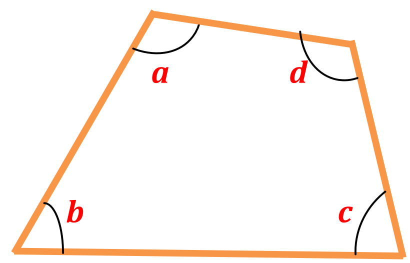


(∠ of equi . Δ)

9.) ANGLE PROPERTIES OF QUADRILATERALS:

a.) The sum of all the angles in a quadrilateral is 360^0.

$a + b + c + d$ (∠ sum of quad .)



b.) The properties of some special quadrilaterals are as follows:

i) **Trapezium:** one pair of parallel opposite sides.

ii) **Isosceles Trapezium**: one pair of parallel opposite sides.

Non – parallel sides are equal in length.

iii) **Parallelogram**: Two pairs of parallel opposite sides.

Opposite sides are equal in length.

Opposite angles are equal.

iv) **Rectangle**: Two pairs of parallel opposite sides.

Opposite sides are equal in length.

All four angles are right angles(90^0) .

Diagonals are equal in length.

Diagonals bisect each other.

V) **Rhombus**: Two pairs of parallel opposite sides.

Four equal sides.

Opposite angles are equal.

Diagonals bisect each other at right angles.

Diagonals bisect the interior angles.

vi)**Square**: Two pairs of parallel opposite sides.

Four equal sides.

All four angles are right angles. (90^0)

Diagonals are equal in length.

Diagonals bisect each other at right angles.

Diagonals bisect the interior angles.

vii) **Kite**: No parallel sides.

Two pairs of equal adjacent sides.

One pair of equal opposite angles.

Diagonals intersect at right angles .

One diagonals bisect the interior angles .

Key Points:

1.) A rectangle with 4 equal sides is a square.
2.) A Parallelogram with 4 right angles is a rectangle.
3.) A parallelogram with 4 equal sides is a rhombus.

4.) A rhombus with 4 equal angles is a square.