COORDINATE GEOMETRY

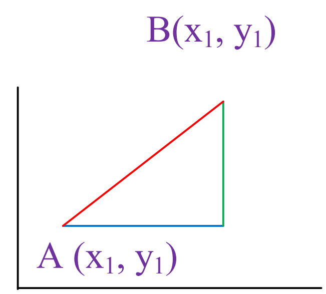
➢ BASIC CONCEPTS AND FORMULAE

I. Length of a Line Segment:

The distance between two points *A* (x_1, y_1) *B* (x_2, y_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To find the length of a line segment joining two points A(x_1, y_1) and B(x_2, y_2), use the formula:

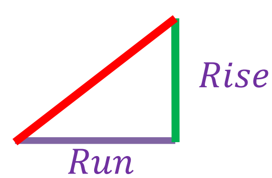


$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

II. Gradient of a Straight Line:

1. **The gradient of a straight line is a measure of its steepness or slope.**
2. The gradient is the ratio of the vertical distance (the rise) to the horizontal distance (the run).

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}}$$

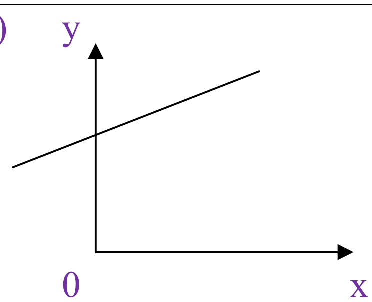


3.To find the gradient m, of the line passing through two points. A (x_1, y_1) and B (x_2, y_2), use the formula:

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

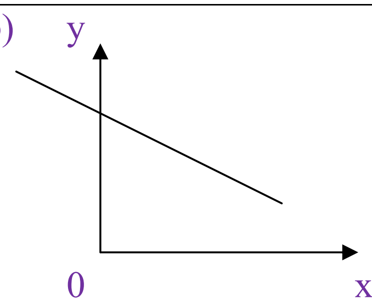
4. **The gradients of some straight lines are given below.**

(a)



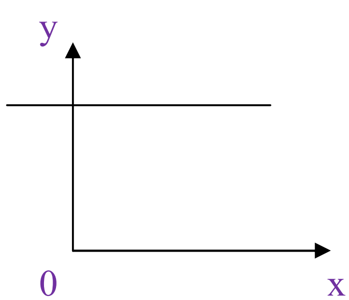
A line sloping upwards to the right has a **positive** gradient

(b)



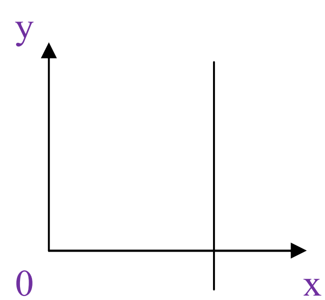
A line sloping upwards to the left has a **negative** gradient.

(c)



The gradient of a horizontal line parallel to the x-axis is **zero.**

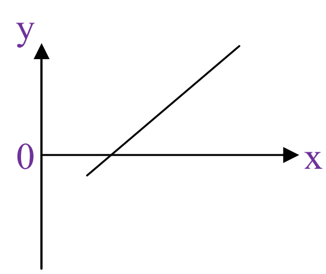
(d)



The gradient of a horizontal line parallel to the y-axis is **undefined.**

5. When three or more points lie on the same straight line, they are **collinear.**
Three points A, B and C are collinear if

Gradient of AB = Gradient of BC = Gradient of AC



III. EQUATION OF A STRAIGHT LINE

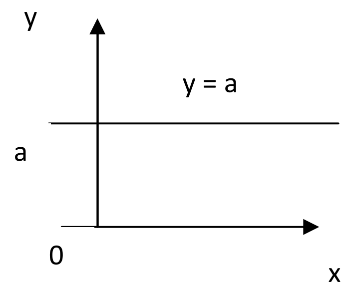
1. To find the equation of a straight line, using the formula:

$$y = mx + c$$

2.The equation of a straight line that is parallel to the x-axis is given by

$$y = a$$

where a is the y- intercept.



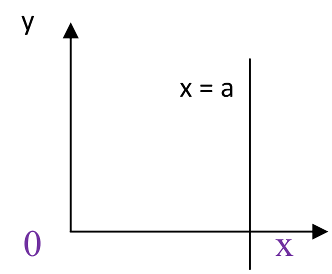
y = 0 is the equation of the x-axis.

3.The equation of a straight line that is parallel to eh y-axis is given by

$$x = a$$

where a is the x- intercept.

x = 0 is the equation of the y-axis



4. **The distance of a point (x , y) from the origin (0, 0) is** $\sqrt{(x^2 + y^2)}$.

5. **The coordinates of a point on *x–axis* is taken as (x, 0) while on *y–axis* it is taken as (0, y) respectively .**

6. **SECTION FORMULA:**

The coordinates of the point *P* (x, y) which divides the line segment joining

A **(x_1, y_1) and *B* (x_2, y_2) internally in the ratio m : n are given by**

$$x = \frac{mx_2 + nx_1}{m+n} \quad \frac{my_2 + ny_1}{m+n}$$

7. **MID – POINT FORMULA:**

Coordinates of mid – point of *A B , where A* (x_1, y_1) and *B* (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

8. **CENTROID OF A TRIANGLE AND ITS COORDINATES:**

The medians of a triangle are concurrent . Their point of concurrence is called the centroid . It divides each median in the ratio 2 : 1. The coordinates of centroid of a triangle with vertices *A* (x_1, y_1) ,*B* (x_2, y_2) and *C* (x_3, y_3) are given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

9. **The area of the triangle formed by the points (x_1 , y_1) ,(x_2, y_2) and (x_3, y_3)is given**

$$\Delta = \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

Example1:

Find the distance between the points

(a) P (2,5) and Q (6, 8) (b) R (-1, 3) and S (7, -4)

Solution:

(a) PQ = $\sqrt{(6\ -\ 2\,)^2\ +\ (8\ -\ 5\,)^2}$
$\sqrt{(4\,)^2\ +\ (3\,)^2}$
$\sqrt{25}$
5 units

(b) RS = $\sqrt{[7 - (-1)]^2\ +\ (-4\ -\ 3\,)^2}$
$\sqrt{(8)^2\ +\ (-3\,)^2}$
$\sqrt{113}$
10.6 units

Example2:

The distance between the points A (20a,-3) and B (a+1, -1) is $\sqrt{13}$ units. Find the possible values of a.

Solution:

AB = $\sqrt{13}$ units

$\sqrt{[(2a+1) - 20a)]^2\ +\ [-1-(-3)]^2}\ = \sqrt{13}$

$(1\text{-}a)^2\ +\ 2^2\ =\ 13$
$1 - 2a + a^2 + 4 = 13$
$a^2 - 2a - 8 = 0$
$(a+2)\,(a-4)\ =\ 0$
$\therefore a + 2\ =\ 0$ or $a - 4 = 0$
a = -2 **or** **a = 4**

Example3:

(a) Find the gradient of the line joining the points P(2, 5) and Q(4, 9)

(b) Find the value of p if te line joining the points R(-3, p) and S(2p, 8) has a gradient of 2.

Solution:

(a) Gradient of PQ $= \frac{9-5}{4-2}$

$= \frac{4}{2}$

$= 2$

(b) Gradient of RS = 2

$\frac{8-p}{2p-(-3)} = 2$

$\frac{8-p}{2p+3} = 2$

Multiply both sides by 2p + 3 →

$8 - p = 2(2p + 3)$

$8 - p = 4p + 6$

$5p = 2$

$\mathbf{P = \frac{2}{5}}$

Example4:

Find the gradient of the line joining the points

(a) A (5, 9) and B(5, -4), (b) C(10, -3) and D(18, -3)

Solution:

(a) Gradient of AB

$= \frac{-4-9}{5-5}$

$= -\frac{13}{0}$

= undefined

(b) Gradient of CD

$= \frac{-3-(-3)}{18-10}$

$= \frac{0}{8}$

$= 0$

Example5:

Given the gradient ,m and the y-intercept, c.

Find the equation of the straight line

(a) That passes through the point (0, 8) and has a gradient of 3.
(b) That passes through the origin and has a gradient of -2

Solution:

(a) $m = 3, \ c = 8$
Equation of line: $y = 3x + 8$

Use $y = mx + c$

(b) $m = -2, c = 0$
Equation of line $y = -2x + 0$
$Y = -2x$

Use $y = mx + c$

Example6:

Given the gradient, m and a point (x_1, y_1)

Find the equation of the straight line that passes through the point (4, 6) and has a gradient of 3.

Solution:

$m = 3$, (4, 6)
equation of line : $y = 3x + c$
at (4, 6),
$6 = 3(4) + c$
$6 = 12 + c$
$C = -6$
$\therefore \mathbf{y = 3x - 6}$

Example7:

Given two points, (x_1, y_1) and (x_2, y_2)

Find the equation of the straight line that passes through the points A(3, -1) and B (5, 7).

Solution:

A (3, -1), B (5, 7)

$$m = \frac{7-(-1)}{5-3}$$
$$= \frac{8}{2}$$
$$= 4$$

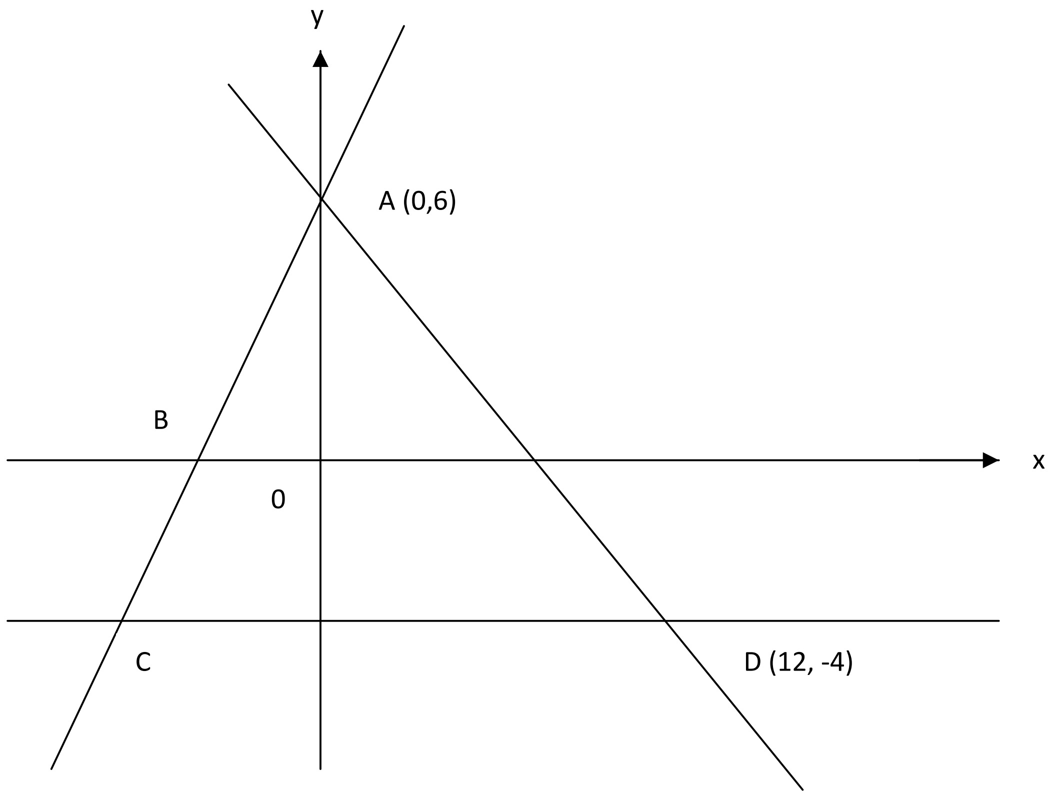
Equation of line: y = 4x + c
At (3, -1),
-1 = 4(3) + c
-1 = 12 + c
C = -13

∴ y = 4x - 13

Example8:

In the diagram, A is the point (0, 6) and D is the point (12, -4). The point B lies on the x-axis. The straight line AB produced meets the horizontal line CD at C.

(a) Find the equation of AD
(b) Find the equation of CD.
(c) Given the equation of the line AB is 2y - 3x = 12
 i) Find the gradient of the line AB,
 ii) The coordinates of B and C
 iii) The length of BC
 iv) The equation of the straight line l which has the same gradient as the line AB and passes through the point D.



Solution:

(a) A = (0, 6), D = (12, -4)

Gradient of AD

$$= \frac{-4-6}{12-0}$$

$$= \frac{-10}{12}$$

$$= \frac{-5}{6}$$

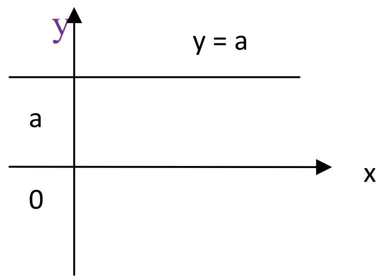
Equation of AD: $y = \frac{-5}{6}x + 6$

(b) **Equation of CD :** $y = -4$

Tip for Students:

The equation of a straight line that is parallel to the x-axis is in the form where a is the y- intercept.

y = a



(c) (i) $2y - 2x = 12$

$2y = 3x + 12$

$y = \frac{3}{2}x + 6$

$y = 1\frac{1}{2}x + 6$

$\therefore$ gradient of AB = $1\frac{1}{2}$

(ii) $y = 1\frac{1}{2}x + 6$

At the x- axis, $y = 0$

$0 = 1\frac{1}{2}x + 6$

$1\frac{1}{2}x = -6$

$x = \frac{-6}{1\frac{1}{2}} = \frac{-6\times2}{3}$

$x = -4$

$\therefore$ B = (- 4, 0)

At the point C, $y = -4$

$-4 = 1\frac{1}{2}x + 6$

$1\frac{1}{2}x = -10$

$x = \frac{-10}{1\frac{1}{2}} \frac{-10\times2}{3}$

$= -6\frac{2}{3}$

$\therefore$ C = $(-6\frac{2}{3}, -4)$

(iii) B = (- 4, 0), C = (- 6 $\frac{2}{3}$,- 4)

$$BC = \sqrt{[-6\tfrac{2}{3}, -(-4)]^2 + (-4-0)^2}$$

$$= \sqrt{(-2\tfrac{2}{3})^2 + (-4)^2}$$

$$= \sqrt{23\tfrac{1}{9}}$$

= 4.81 units

(iv) Gradient of l = Gradient of AB

$= 1\frac{1}{2}$

Equation of l : $y = 1\frac{1}{2}x + c$

At the point D (12 , -4),

$-4 = 1\frac{1}{2}(12) + c$

$-4 = 18 + c$

C = -22

$\therefore\ y = 1\frac{1}{2}x - 22$

Example10:

A straight line l passes through the point (1, 10) and has gradient 2.

(a) Write down the equation of the line l.
(b) Given that the straight line l also passes through the point (k , 3k + 3), find the value of k.
(c) The straight line l intersects the x-axis at the point P and the y-axis at the point Q. find the coordinates of P and Q.
(d) Find the length of PQ.
(e) Find the perpendicular distance from the origin to the line l.

Solution:

(a) Equation of l: $y = 2x + c$

At (1, 10),
$10 = 2(1) + c$
$C = 8$
$\therefore$ **$y = 2x + 8$**

(b) Since the point (k, 3k + 3) lies on the line $y = 2x + 8$, the coordinates (k, 3k + 3) must satisfy the equation.
Substitute (k, 3k + 3) into $y = 2x + 8$,
$3k + 3 = 2(k) + 8$
$3k - 2k = 8 - 3$
$\therefore$ **$k = 5$**

(c) $y = 2x + 8$
At the x-axis, $y = 0$,
$0 = 2x + 8$
$2x = -8$
$x = -4$
$\therefore$ **P = (-4, 0)**

$y = 2x + 8$
y – intercept = 8
$\therefore$ Q = (0, 8)

Use y = mx + c

Where m = gradient

And c = y intercept.

(d) P = (-4, 0) , Q (0, 8)

$$PQ = \sqrt{[0 - (-4)]^2 + (8 - 0)^2}$$

$$= \sqrt{(4)^2 + (8)^2}$$

$$= \sqrt{80}$$

= 8.94 units

EXAMPLE11:

1. **Show that points A(-3, 5) B(3, 1), C (0, 3) and D (-1, -4) do not form a quadrilateral.**

Solution:

$$AB = \sqrt{(3+3)^2 + (1-5)^2}$$

$$= \sqrt{52} \qquad = 2\sqrt{13}$$

$$BC = \sqrt{(0-3)^2 + (3-1)^2}$$

$$= \sqrt{9+4} \qquad = \sqrt{13}$$

$$CA = \sqrt{(-3-0)^2 + (5-3)^2}$$

$$= \sqrt{9+4} \qquad = \sqrt{13}$$

Then, AB = BC + CA $\qquad$ $\because \mathbf{2\sqrt{13} = \sqrt{13} + \sqrt{13}}$

$\therefore$ The points A, B, C lie on a line. It means collinear and hence A, B,C,D do not form a quadrilateral.

EXAMPLE12:

2. The vertices of a triangle are (1, k) (4, -3) and (-9,7), if the area of the triangle is 24 sq. units then find value of k.

Solution:

By given data:

Area of triangle = 24 Sq. units

$$\therefore \left| \begin{array}{cc} 1 & k \\ 4 & -3 \\ -9 & 7 \\ 1 & k \end{array} \right| = 24$$

| (-3-4k) + (28 – 27) + (-9k – 7) | = 48

| -3 -4k + 1 – 9k – 7| = 48

| -13k – 9| = 30

-13k – 9 = ±48

-13 k – 9 = 48 or -13 k – 9 = - 48

-13k = 57 or -13k = - 39

$\therefore$ k = $\frac{-57}{13}$ or k = 3

EXAMPLE13:

3. Points P, Q, R and S in that order divides line segment joining points A(2, 5) and B(7, -5) in five equal parts. Find the co-ordinates of P, Q, R, and S.

Solution:

Clearly, point Q(x, y) divide the line segment AB in ratio 2 : 3

$\therefore\ x = \frac{2 \text{ x } 7+3 \text{ x } 2}{2+3}$ and $y = \frac{2 \text{ x } (-5)+ 3 \text{ x } 5}{2+3}$

$x = \frac{14+6}{5} = 4$ and $y = \frac{-10+15}{5} = 1$

$\therefore$ Coordinates of point Q are (4, 1)

Point R divides AB in ratio 3 : 2

By section formula , $x = \frac{3 \text{ x } 7+2 \text{ x } 2}{2+3} = \frac{21+4}{5} = \frac{25}{5} = 5$

$y = \frac{3 \text{ x } (-5)+ 2 \text{ x } 5}{3+2} = \frac{-15+10}{5} = \frac{-5}{5} = -1$

$\therefore$ Coordinates of point R are (5, -1)

Now, P is mid point of line segment AQ

∴ Coordinates of point P are ($\frac{2+4}{2},\frac{5+1}{2}$) i.e., P(3, 3)

S is the mid point of the line segment RB

∴ Coordinates of point S are ($\frac{5+7}{2},\frac{-1-5}{2}$) i.e., S (6, -3)

EXAMPLE14:

4. Find the coordinates of point P on AB such that $\frac{PA}{PB} = \frac{3}{4}$ where A (3, 1) and B (-2, 5)

Solution:

By given data $\frac{PA}{PB} = \frac{3}{4}$ → PA : PB = 3 : 4

Let P (x, y) divide the line segment AB.

By section formula,

$$x = \frac{3 \text{ x } (-2) + 4 \text{ x } 3}{3+4} \text{ and}$$

$$y = \frac{3 \text{ x } 5 + 4 \text{ x } 1}{3+4}$$

i.e., $x = \frac{-6+12}{7} = \frac{6}{7}$

and $y = \frac{15+4}{7} = \frac{19}{7}$

∴ co-ordinates of point P are ($\frac{6}{7}, \frac{19}{7}$)

➢ TYPICAL PROBLEMS

TYPE A : PROBLEMS BASED ON DISTANCE FORMULA:

1. **Find the distance between the following pairs of points :**

i) (- 5, 7) , (-1, 3)   **ii) (a, b) , (-a , -b)**

SOLUTION:

i) **Let two given points be *A* (- 5, 7) and *B* (-1, 3).**

Thus, we have $x_1 = -5$ and $x_2 = -1$

$y_1 = 7$, and $y_2 = 3$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \quad AB = \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16}$$

$$= \sqrt{32} = 4\sqrt{2}$$

ii) **Let two given points be *A* (a, b) and *B* (-a , -b)**

Hear, $x_1 = a$ and $x_2 = -a$

$y_1 = b$, and $y_2 = -b$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\Rightarrow$ ***A B*** = $\sqrt{(-a-a)^2+(-b-b)^2}$

= $\sqrt{(-2a)^2+(-2b)^2}$

= $\sqrt{4a^2+4b^2}$ = $2\sqrt{a^2+b^2}$.

2. **Determine the points (1,5), (2, 3) and (-2, -11) are collinear .**

SOLUTION:

Let *A* (1, 5) , *B* (2, 3) *C* (-2, -11) be given points . then, we have

A B = $\sqrt{(2-1)^2+(3-5)^2}$ = $\sqrt{1+4}$ = $\sqrt{5}$

BC = $\sqrt{(-2-2)^2+(-11-3)^2}$ = $\sqrt{16+196}$ = $\sqrt{4X53}$ = $2\sqrt{53}$

A C = $\sqrt{(-2-1)^2+(-11-5)^2}$ = $\sqrt{9+256}$ = $\sqrt{265}$

Clearly, ***AB*** **+** ***BC*** $\neq$ ***AC***

$\therefore$ ***A , B and C*** **are not collinear .**

3. **Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.**

SOLUTION:

Let *A* (5, -2), *B* (6, 4) and *C* (7, -2) be the vertices of a triangle .

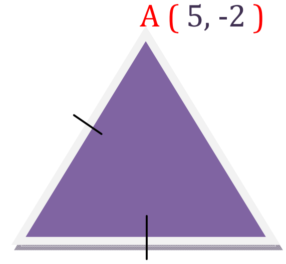
Then, we have

$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$

$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{4} = 2$

Here, $AB = BC$



$\therefore$ **Δ *ABC* is an isosceles triangle.**

4. **Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer :**

i) (-1, -2) , (1, 0), (-1,2), (-3 ,0) **ii) (4, 5), (7, 6), (4, 3), (1, 2)**

SOLUTION:

i) **Let *A*(-1, -2) , *B*(1, 0), *C*(-1,2), *D*(-3 ,0) be the four given points .**

Then, using distance formula, we have

$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2 + (0-2)} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$DA = \sqrt{(-1+3)^2+(-2+0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AC = \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{0+16} = \sqrt{4}$

And $BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{16} = 4$

Hence, four sides of quadrilateral are equal and diagonal *AC* and *BD* are also equal.

$\therefore$ **Quadrilateral *ABCD* is square.**

ii) **Let *A* (4, 5), *B* (7, 6), *C* (4, 3), and *D* (1 ,2) be the four given points .**

Then, using distance formula, we have

$AB = \sqrt{(7-4)^2+(6-5)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2+(3-6)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(1-4)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$DA = \sqrt{(4-1)^2+(5-2)^2} = \sqrt{9+9} = \sqrt{18} = \sqrt{2}$

$AC = \sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+4} = 2$

And $BD = \sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Clearly, *AB* = *CD*, *BC* = *DA* and *AC* $\neq$ *BD*

∴ *ABCD* is a parallelogram .

5. Find the point on the x – axis which is equidistant from (2,-5) and (-2, 9) .

SOLUTION:

Let *P* (x, 0) be any point on x - axis .

Now, *P* (x, 0) is equidistant from point *A* (2, -5) and *B* (-2, 9)

∴ *AP* = *BP*

$$\Rightarrow \sqrt{(x-2)^2+(0+5)^2} = \sqrt{(x+2)^2+(0-9)^2}$$

Squaring both sides, we have

$(x-2)^2 + 25 = (x+2)^2 + 81$

$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$

$\Rightarrow -8x = 56$

∴ $x = \frac{56}{-8} = -7$

∴ The point on the x – axis equidistant from given point is (- 7, 0)

6. Find the relation between *x* and *y* such that the point (*x* , *y*) is equidistant from the point (3, 6) and (-3, 4)

SOLUTION:

Let *P* (x, y) be equidistant from the points *A* (3, 6) and *B* (-3, 4)

i.e., *PA* = *PB*

Squaring both sides , we get

$\Rightarrow$ $AP^2 = BP^2$

$\Rightarrow$ $(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$\Rightarrow$ $x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$\Rightarrow$ $-12x - 4y + 20 = 0$ $\Rightarrow$ $3x + y - 5 = 0$

7. Find the relation between *x* and *y* if the point (*x* , *y*) , (1, 2) and (7, 0) are collinear .

SOLUTION:

Given points are *A* (x, y), *B* (1, 2) and *C* (7, 0)

These points will be collinear only if the area of the triangle formed by them is zero, i.e.

$$\text{Area of } (\Delta ABC) = \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) 1]$$

$$\Rightarrow 0 = \frac{1}{2} [x (2 - 0) + 1 (0 - y) + 7 (y - 2)]$$

$$\Rightarrow 2x + 6y - 14 = 0$$

$$\Rightarrow x + 3y = 7$$

TYPE B : **PROBLEMS BASED ON DISTANCE FORMULA:**

1. Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3 .

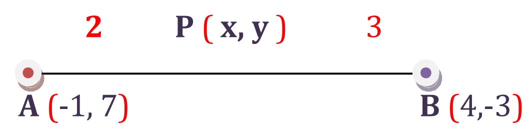
SOLUTION:

Let (x, y) be the required point . Thus, we have

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

Therefore,

$$x = \frac{2 \text{ X } 4 + 3 \text{ X } (-1)\ 1}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$



And , $$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$y = \frac{2 \text{ X } (-3) + 3 \text{ X } 7}{2+3} = \frac{6+21}{5} = \frac{15}{5} = 3$$

So , the coordinates of Pare (1, 3).

2. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

SOLUTION:

Let the given points *A* (4, -1) and *B* (-2 , -3) and points of trisection be *P* and *Q* .

Let $AP = PQ = QB = k$

$\therefore\ PB = PQ + QB = k + k = 2k$

$AP : PB = k : 2k = 1 : 2$

Therefore, coordinates of *P* are

$$\left(\frac{1 \text{ X } -2 + 2 \text{ X } 4}{3}, \frac{1 \text{ X } -3 + 2 \text{ X } -1}{3} \right) = \left(2, \frac{5}{3}\right)$$

Now $AQ = AP + PQ = k + k = 2k$

$\therefore AQ : QB = 2k : k = 2 : 1$

And, coordinates of Q are

Therefore, coordinates of P are

$$\left(\frac{2 \times -2 + 1 \times 4}{3}, \frac{2 \times -3 + 1 \times -1}{3}\right) = \left(0, -\frac{7}{3}\right)$$

Hence , points of trisection are $= \left(2, \frac{5}{3}\right)$ and $\left(0 - \frac{7}{3}\right)$

3. Find the ratio in which the line segment joining A (1, -5) and B (- 4, 5) is divided by the x - axis. Also find the coordinates of the point of division.

SOLUTION:

Let the required ratio be k : 1. Then co-ordinates of the point of division is

$$P = \left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$$

Since, this point lies on x-axis. Then, its y-coordinate is zero.

i.e. $\frac{5k-5}{k+1} = 0$

$\Rightarrow 5k - 5 = 0 \quad \Rightarrow 5k = 5$

$\Rightarrow k = \frac{5}{5} = 0$

Thus, the required ratio is **1 : 1** and the point of division is $P = \left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$

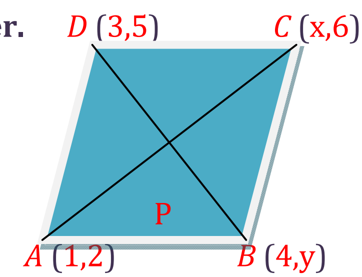
ie., $P = \left(\frac{-3}{2}, 0\right)$

4. **If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order. Find *x* and *y*.**

SOLUTION:

Let *A*(1, 2), *B* (4, y),*C* (x, 6) and *D*(3, 5)be the vertices of a parallelogram *ABCD*.

Since , the diagonals of a parallelogram bisect each other.



$$\therefore \left(\frac{X+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+Y}{2}\right)$$

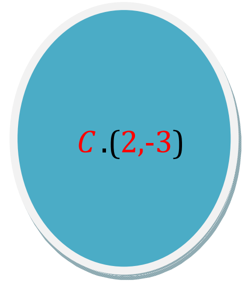
$$\Rightarrow \frac{X+1}{2} = \frac{7}{2} \quad \Rightarrow x+1 = 7 \quad \therefore x = 6$$

And , $4 = \frac{5+Y}{2} \quad \Rightarrow 5+y = 8 \quad \therefore y = 8-5 = 3$

Hence, $x = 6$ **and** $y = 8-5 = 3$

5. **Find the coordinates of a point *A*, where *AB* is the diameter of a circle whose**

center is (2, -3) and *B* is (1, 4).

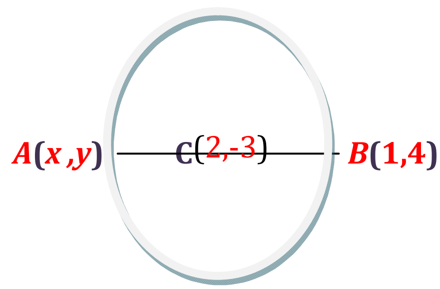


SOLUTION:

Let the coordinates of *A* be (*x* ,*y*).

Now, *C* is the center of circle therefore, the coordinates of

$C = \left(\frac{X+1}{2} , \frac{6y+4}{2} \right)$ but coordinates of *C* are given (2, -3).



$\therefore \quad \frac{X+1}{2} = 2 \quad \Rightarrow \quad x + 1 = 4 \quad \therefore \quad x = 3$

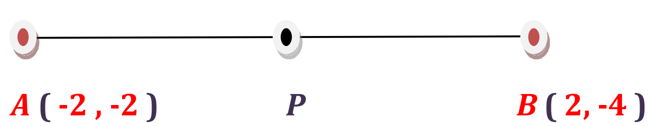
And, $y = \frac{Y+4}{2} = -3 \quad \Rightarrow \quad y + 4 = -6 \quad \therefore \quad y = -10$

Hence, coordinates of *A* (3, -10).

6. If *A* and *B* are (-2, -2) and (2, -4), respectively, find the coordinates of *P* such that $AP = \frac{3}{7} AB$ and *P* lies on the line segment *AB*.

SOLUTION:

We have, $AP = \frac{3}{7} AB$



$\Rightarrow \quad \frac{AP}{AB} = \frac{3}{7} \qquad \Rightarrow \quad \frac{AB}{AP} = \frac{7}{3}$

$\Rightarrow \quad \frac{AP + PB}{AP} = \frac{7}{3} \qquad \Rightarrow \quad \frac{AP}{AP} + \frac{PB}{AP} = \frac{7}{3}$

$\Rightarrow \quad 1 + \frac{PB}{AP} = \frac{7}{3} \qquad \Rightarrow \quad \frac{PB}{AP} = \frac{7}{3} - 1 = \frac{4}{3}$

$\Rightarrow \quad \frac{AP}{AB} = \frac{3}{4}$

$\Rightarrow \quad AP : PB = 3 : 4$

Let P (x, y) be the point which divides the join of (-2, -2) and B(2, -4) in the ratio 3 : 4

$$\therefore \quad x = \frac{3 \text{ X } 2 + 4 \text{ X } -2}{3 + 4} = \frac{6 - 8}{7} = \frac{-2}{7}$$

$$y = \frac{3 \text{ X } -4 + 4 \text{ X } -2}{3 + 4} = \frac{-12 - 8}{7} = \frac{-20}{7}$$

Hence, the coordinates of the point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

7. Find the coordinates of the points which divide the line segment joining A (-2, 2) and B (2, 8) into four equal parts .

SOLUTION:

Let *P, Q, R* be the points hat divide the line segment joining *A* (-2, 2) and *B* (2, 8)

into four equal parts .

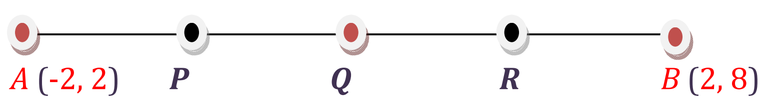
Since, *Q* divides the line segment *AB* into two equal parts i.e., *Q* is mid-point of *AB* .

$\therefore$ Coordinates of *Q* are $\left(\frac{-2+2}{2} , \frac{2+8}{2} \right)$ i.e., (0, 5)

Now, *P* divides *AQ* into two equal parts i.e., *P* is the mid-point of *AQ* .

$\therefore$ Coordinates of *P* are $\left(\frac{-2+0}{2} , \frac{2+5}{2} \right)$

i.e., $\left(-1 + \frac{7}{2} \right)$.



Again, *R* is the mid – point of *QB* .

$\therefore$ Coordinates of *R* are $\left(\frac{0+2}{2} , \frac{5+8}{2} \right)$

i.e., $\left(1 + \frac{13}{2} \right)$.

8. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1)

taken in order.

SOLUTION:

Let A (3, 0), B (4, 5), C (-1, 4) and D (-2, -1) be the vertices of a rhombus,

Therefore, its diagonals

$$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$BD = \sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$\therefore$ Area of rhombus $ABCD$ = $\frac{1}{2}$ X (Product of length of diagonals)

$$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24 \text{ sq.units}.$$

TYPE C : PROBLEMS BASED ON AREA OF TRIANGLE:

1. **Find the area of the triangle whose vertices are :**

   (-5,-1), (3, -5), (5, 2)

SOLUTION:

Let $A(x_1, y_1) = (-5, -1)$

$B\ (x_2, y_2) = (3, -5)$

$C\ (x_3, y_3) = (5, 2)$

$\therefore$ Area of $\Delta\ ABCD = \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2} \times [-5(-5 - 2) + 3(2 + 1) + 5(-1 + 5)]$

$= \frac{1}{2} \times (35 + 9 + 20) = \frac{1}{2} \times 64 = 32$ sq. units .

2. **Find the value of 'k', for which the points are collinear $(7, -2)$, $(5, 1)$, $(3, k)$.**

SOLUTION:

Let the given points be

$A\ (x_1, y_1) = (7, -2)$

$B\ (x_2, y_2) = (5, 1)$

$C\ (x_3, y_3) = (3, k)$

Since these points are collinear therefore area ($\Delta\ ABC$)

$\Rightarrow \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 7(1 - k) + 5(k + 2) + 3(-2 - 1) = 0$

$\Rightarrow 7 - 7 - k + 5 + 10 - 9 = 0$

$\Rightarrow$ $-2k + 8 = 0$

$\Rightarrow$ $2k = 8$

$\Rightarrow$ $k = 4$

Hence, given points are collinear for $k = 4$

3. **Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2,1) and (0,3). Find the ratio of this area of the given triangle .**

SOLUTION:

Let $A(x_1, y_1) = (0, -1)$

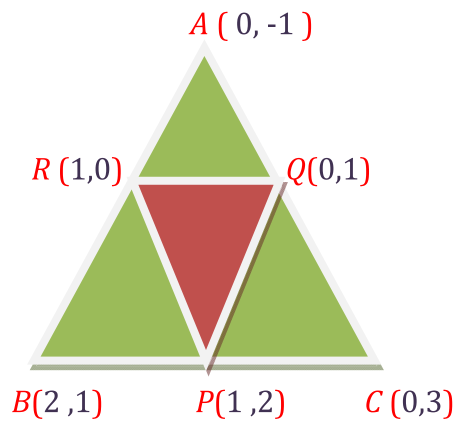
$B(x_2, y_2) = (2, 1)$

$C(x_3, y_3) = (0, 3)$

Be the vertices of ΔABC.

Now, let *P, Q, R* be the mid-points of *BC, CA* and *AB* respectively.

So coordinates of *P, Q, R* are



$$P = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$Q = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)\ )$$

$R = \left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$

Therefore, Area of (Δ *PQR*) = $\frac{1}{2}$ [1 (1 – 0) + 0 (0 – 2) + 1 (2 – 1)] = 0

= $\frac{1}{2}$ (1 + 1) = 1 sq. units

Now, **Area of (Δ *ABC*) = $\frac{1}{2}$ [0 (1 – 3) + 2 (3 + 1) + 0 (- 1 – 1)]**

= $\frac{1}{2}$ [0 X - 2 + 8 + 0] = $\frac{8}{2}$ =4 sq. units

∴ **Ratio of area (Δ *PQR*) to the area (Δ *ABC*) = 1 : 4 .**

4. **A median of a triangle divides it into two triangles of equal areas. Verify this results for Δ *ABC* whose vertices are *A* (4, -6), *B* (3 ,-2) and *C* (5, 2).**

SOLUTION:

Since *AD* is the median of Δ *ABC*, therefore, *D* is the mid-point of *BC* .

Coordinates of *D* are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ i.e., (4, 0)

Now , Area of (Δ *ABD*) = $\frac{1}{2}$ [4 (- 2 – 0) + 3 (0 + 6) + 4 (- 6 + 2)]

= $\frac{1}{2}$ (- 8 + 18 - 16) = $\frac{1}{2}$ X (- 6) = - 3

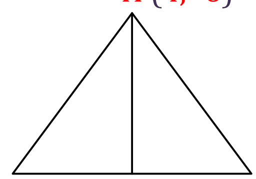
Since area is a measure, which cannot be negative.

Therefore, ar $(\Delta ABD) = 3$ sq. units

And, Area of $(\Delta ADC) = \frac{1}{2}[4(0-2)+4(2+6)+5(-6-0)]$

$$= \frac{1}{2}(-8+32-30) = \frac{1}{2} \times (-6) = -3$$

$\therefore$ area of $(\Delta ADC) = 3$ sq. units



Hence, area (ΔABD) = area (ΔADC)

Hence, the median divides it into two triangles of equal areas.

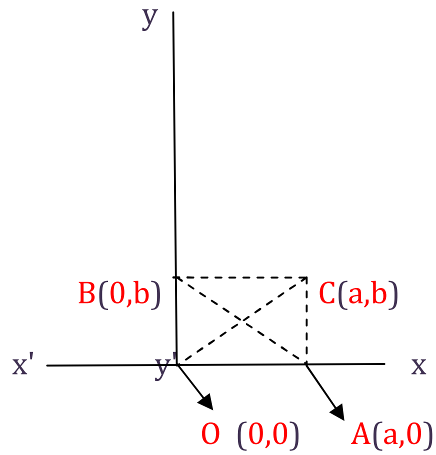
HOTS (HIGHER ORDER THINKING SKILLS PROBLEMS)

1. Prove that the diagonals of a rectangle bisect each other and are equal.

SOLUTION:

Let *OACB* be a rectangle such that *OA* is along x - axis and ob is along y - axis. Let *OA* = a and *OB* =b

Then, the coordinates of *A* and *B* are (a, 0) and (0, b) respectively.



Since, *OACB* is a rectangle.

Therefore, $AC = OB$

$\Rightarrow AC = b$

Thus, we have

$OA = a$

$BC = a$

So, the coordinates of C are (a, b).

The coordinates of the mid-points of *OC* are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Also, **the coordinates of the mid-points of *AB* are** $\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$

Clearly, coordinates of the mid-points of *AB* are same.

Hence, *OC* and *AB* bisect each other.

Also , $OC = \sqrt{a^2+b^2}$ **and** $BD = \sqrt{(a-0)^2+(0-b)^2} = \sqrt{a^2+b^2}$

$OC = AB$

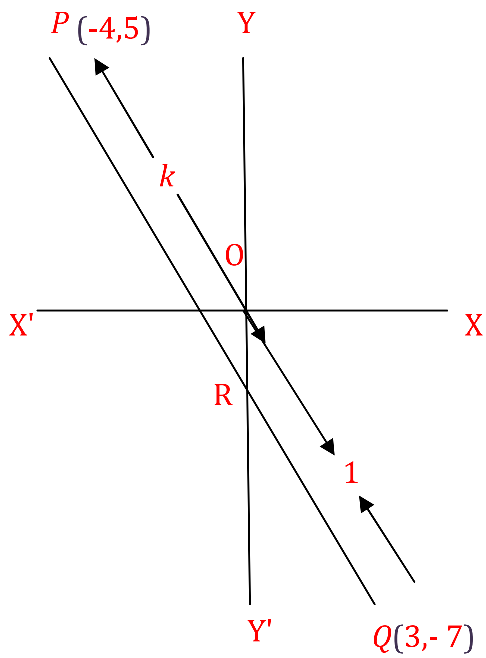
2. **In what ratio does the y- axis divide the line segment joining the point *P* (-4, 5) and *Q* (3, - 7) ? Also, find the coordinates of the point of intersection.**

SOLUTION:

Suppose y - axis divides PQ in the ratio k : 1 .

Then, the coordinates of the point of division are

$$R = \left(\frac{3k-4}{k+1}, \frac{-7k+5}{k+1} \right)$$



Since , R lies on y - axis and x - coordinate of every Point on y - axis is zero.

$$\therefore \quad \frac{3k-4}{k+1} = 0$$

$$\Rightarrow \quad 3k - 4 = 0$$

$$k = \frac{4}{3}$$

Hence, the required ratio is $\frac{4}{3}$: 1 i.e. 4 : 3 .

Putting $k = 4/3$ in the coordinates of R, we find

its coordinates are $\left(0, \frac{-13}{7}\right)$.

3. The line joining the points (2, 1) and (5, -8) is trisected by the points P and Q . If the point P lies on the line $2x - y + k = 0$, find the value of k .

SOLUTION:

As line segment *AB* is trisected by the points *P* and *Q*.

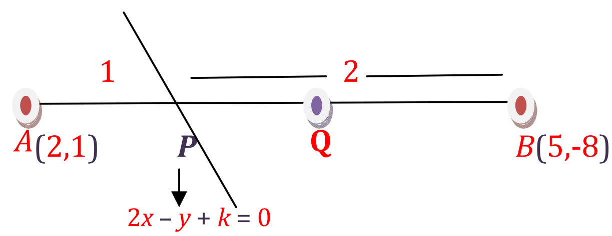
Therefore, Case I : When *AP* : *PB* = 1 : 2 .

$\therefore$ **Coordinates of *P* are**

$$\left\{\frac{1X5+2X2}{1+2},\frac{1X-8+1X2}{1+2}\right\}$$

$\Rightarrow$ ***P* (3, -2)**

Since the point *P* (3,-2) lies on the line



$\Rightarrow$ 2 X 4 - (-5) + k = 0

$\Rightarrow$ k = - 13

$\Rightarrow$ $2x - y + k = 0$

$\Rightarrow$ 2 X 3 - (-2) + k = 0

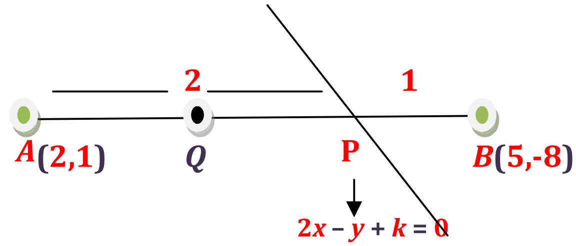
$\Rightarrow$ k = - 8

Case II : When *AP* : *PB* = 2 : 1 .

Coordinates of point *P* are

$$\left\{\frac{2X5+1X2}{2+1},\frac{2X-8+1X1}{2+1}\right\}$$

Since the point *P* (4,-5) lies on the line



$\Rightarrow$ $\mathbf{2x - y + k = 0}$

$\Rightarrow$ **2 X 4 - (-5) + k = 0**

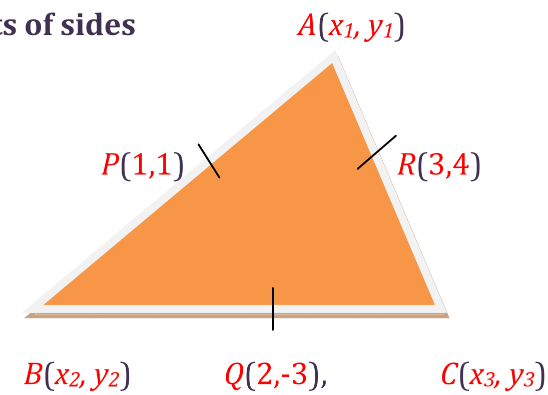
$\Rightarrow$ **k = - 13**

4. **If the coordinates of the mid-points of the sides of the triangle are (1, 1), (2, -3) and (3, 4) . Find the Centroid of the triangle.**

SOLUTION:

Let *P* (1, 1), *Q* (2, -3), *R* (3, 4) be the mid-points of sides *AB, BC* and *CA* respectively of rectangle *ABC* .

Let *A* (x_1, y_1), *B* (x_2, y_2) and *C* (x_3, y_3) be the vertices of triangle *ABC*.



Then, *P* is the mid-point of *AB*.

$$\Rightarrow \quad \frac{x_1 + x_2}{2} = 1 \quad , \quad \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow \quad x_1 + x_2 = 2 \text{ and } y_1 + y_2 \qquad \dots (i)$$

***Q* is the mid-point of *BC* .**

$$\Rightarrow \quad \frac{x_2 + x_3}{2} = 1 \; , \; \frac{y_2 + y_3}{2} = -3 \quad \Rightarrow \quad x_2 + x_3 = 4 \text{ and } y_2 + y_3 = -6 \qquad \dots (ii)$$

***R* is the mid-point of *AC*.** $\Rightarrow \quad x_1 + x_3 = 6 \text{ and } y_1 + y_3 = 8 \qquad \dots (iii)$

$$\Rightarrow \quad \frac{x_1 + x_3}{2} = 3, \quad \frac{y_1 + y_3}{2} = 4$$

From (*i*), (*ii*) and(*iii*), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$$

and $y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$$\Rightarrow \quad \frac{x_1 + x_2 + x_3}{2}, \quad \frac{y_1 + y_2 + y_3}{2} \qquad \ldots . (iv)$$

The coordinates of the centroid of *ABC* are

$$\left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2} \right) = \left(\frac{6}{3}, \frac{2}{3} \right) = \left(2, \frac{2}{3} \right) \qquad \text{[Using equation (} iv \text{)}$$

5.Find the center of a circle passing through the points (6, -6), (3, -7) and (3, 3).

SOLUTION:

Let *O* (*x* ,*y*) be the center of circle. Given points are *A* (6, -6), *B* (3, -7) and *C* (3, 3).

Then , $OA = \sqrt{(x-6)^2 + (y+6)^2}$

$OB = \sqrt{(x-3)^2 + (y+7)^2}$

And , $OA = \sqrt{(x-3)^2 + (y-3)^2}$

Since each point on the circle is equidistant from center.

$\therefore$ $OA = OB = OC =$ Radius

Since $OA = OB$

$$\Rightarrow \sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$$

Squaring both side

$$(x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2$$

$$\Rightarrow x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$$

Or $-6x - 2y = -14$

Or $3x + y = 7$

Similarly,

$OB = OC$

$$\Rightarrow \sqrt{(x-3)^2+(y+7)^2} = \sqrt{(x-3)^2+(y-3)^2}$$

Squaring both side , we get

$$(x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$$

$$\Rightarrow (y+7)^2 = (y-3)^2$$

$\Rightarrow \; y^2 + 14y + 49 = y^2 - 6y + 9$

$\Rightarrow \quad y = -2$

Hence, the coordinates of the center of circle are (3, -2)

6. **Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, -2) and B (3, 7)**

SOLUTION:

Let P (x_1, y_1) is common point of both lines and divides the line segment joining

A(2, -2) and B (3, 7) in ratio k : 1 .

$\therefore \quad x_1 = \dfrac{3k + 2}{k + 1}$,

And , $\quad y_1 = \dfrac{7k + 1(-2)}{k + 1} = \dfrac{7k - 2}{k + 1}$

Since, point (x_1, y_1) lies on the line

$$2x + y = 4$$

$$\therefore \quad 2\left(\frac{3k + 2}{k + 1}\right) + \left(\frac{7k - 2}{k + 1}\right) = 4$$

$$\Rightarrow \quad \frac{6k + 4 + 7k - 2}{k + 1} = 4$$

Or $\quad 13k + 2 = 4k + 4$

Or $9k = 2$

Or $k = \frac{2}{9}$

So, Required ratio is $\frac{2}{9} : 1$ **Or** $2 : 9$

SUMMARY AND KEY POINTS

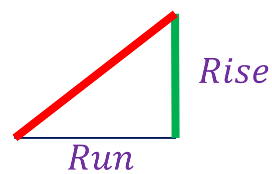
1. **The distance between two points** $A(x_1, y_1)$ $B(x_2, y_2)$ **is given by**

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. **The gradient of a straight line is a measure of its steepness or slope.**

3. The gradient is the ratio of the vertical distance (the rise) to the horizontal distance (the run).

$$\text{Gradient} = \frac{Rise}{Run}$$

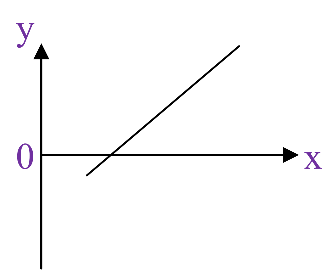


3. To find the gradient m, of the line passing through two points. A (x_1, y_1) and B (x_2, y_2), use the formula:

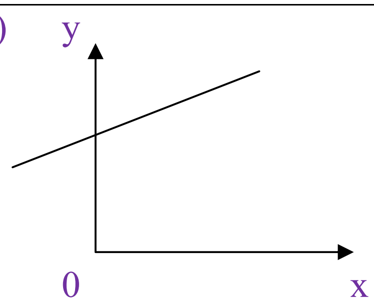
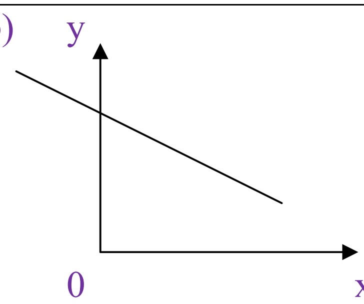
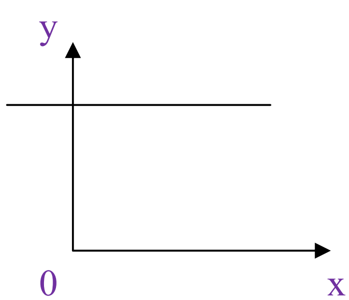
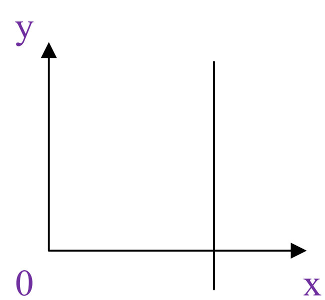
$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

4. When three or more points lie on the same straight line, they are collinear. Three points A, B and C are collinear if

Gradient of AB = Gradient of BC = Gradient of AC



5. **The gradients of some straight lines are given below.**

(a)  A line sloping upwards to the right has a **positive** gradient	(b)  A line sloping upwards to the left has a **negative** gradient.
(c)  The gradient of a horizontal line parallel to the x-axis is **zero.**	(d)  The gradient of a horizontal line parallel to the y-axis is **undefined.**

6**. EQUATION OF A STRAIGHT LINE**

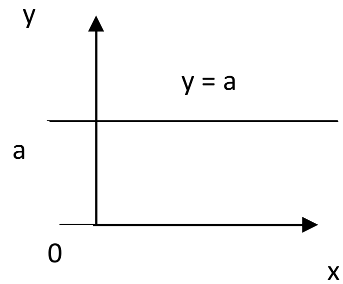
a.) To find the equation of a straight line, using the formula:

$$y = mx + c$$

b.) The equation of a straight line that is parallel to the x-axis is given by

| y = a | where a is the y- intercept.

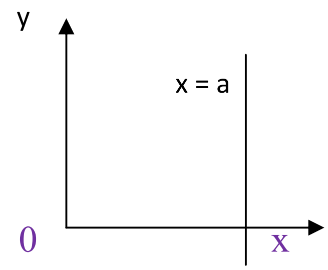
y = 0 is the equation of the x-axis.



c.) The equation of a straight line that is parallel to the y-axis is given by

| x = a | where a is the x- intercept.

x = 0 is the equation of the y-axis



7. **The distance of a point (*x* , *y*) from the origin (0, 0) is** $\sqrt{(x^2+y^2)}$.

8. **The coordinates of a point on *x–axis* is taken as (x, 0) while on *y–axis* it is taken as (0, y) respectively .**

9. **SECTION FORMULA:**

The coordinates of the point *P* (x, y) which divides the line segment joining *A* (x_1, y_1) and *B* (x_2, y_2) internally in the ratio m : n are given by

$$x = \frac{mx_2 + nx_1}{m+n} \quad \frac{my_2 + ny_1}{m+n}$$

10.MID – POINT FORMULA:

Coordinates of mid – point of *A B , where A* (x_1, y_1) and *B* (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

11. **CENTROID OF A TRIANGLE AND ITS COORDINATES:**

The medians of a triangle are concurrent . Their point of concurrence is called the centroid . It divides each median in the ratio 2 : 1. The coordinates of centroid of a triangle with vertices *A* (x_1, y_1) ,*B* (x_2, y_2) and *C* (x_3, y_3) are given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

12. The area of the triangle formed by the points (x_1, y_1) ,(x_2, y_2) and (x_3, y_3)is given

$$\Delta = \frac{1}{2} [x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

13. Tip for Students:

The equation of a straight line that is parallel to the x-axis is in the form where a is the y- intercept.

y = a