Q $\lim_{x \to -\infty} \sqrt{x^2+x} + \sqrt[3]{x^3+6x^2+1}$

$(\infty)\uparrow + (-\infty)\uparrow$ {An Indeterminate Form}

put $x = -y$

& So $y \to \infty$

$\lim_{y \to \infty} \sqrt{y^2-y} + \sqrt[3]{-y^3+6y^2+1}$

taking y common, taking $(-y)$ common

$\lim_{y \to \infty} |y|\sqrt{1-\frac{1}{y}} - y\sqrt[3]{1-\frac{6}{y}-\frac{1}{y^3}}$

$\lim_{y \to \infty} y\left(1-\frac{1}{y}\right)^{\frac{1}{2}} - y\left(1-\frac{6}{y}-\frac{1}{y^3}\right)^{\frac{1}{3}}$

$\lim_{y \to \infty} y\left\{\left(1-\frac{1}{y}\right)^{\frac{1}{2}} - \left(1-\frac{6}{y}-\frac{1}{y^3}\right)^{\frac{1}{3}}\right\}$

Using Binomial approximation

$(1+t)^n \simeq 1+nt$ when $t \to 0$

$\left(1+\left(-\frac{1}{y}\right)\right)^{\frac{1}{2}} \simeq 1 - \frac{1}{2y}$ becoz $\left(-\frac{1}{y}\right) \to 0$ as $y \to \infty$

Similarly $\left\{1+\left(-\frac{6}{y}-\frac{1}{y^3}\right)\right\}^{\frac{1}{3}} \simeq 1 + \frac{1}{3}\left(-\frac{6}{y}-\frac{1}{y^3}\right)$

$\simeq 1 - \frac{2}{y} - \frac{1}{3y^3}$

$\lim_{y \to \infty} y\left\{\cancel{1} - \frac{1}{2y} - \cancel{1} + \frac{2}{y} + \frac{1}{3y^3}\right\}$

$\lim_{y \to \infty} -\frac{1}{2} + 2 + \frac{1}{3y^2} \to (0\uparrow)$

$2 - \frac{1}{2} = \left(\frac{3}{2}\right)$ Ans

Q① Find value of $\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right)$

Q② If $\cos\left(\frac{\pi}{15}\right)\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{3\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{5\pi}{15}\right)\cos\left(\frac{6\pi}{15}\right)\cos\left(\frac{7\pi}{15}\right) = \frac{1}{K}$ then $K = ?$

Q③ Find $\sum_{K=1}^{7} \tan^2\left(\frac{K\pi}{16}\right)$

Q④ Find the Sum of all the possible solutions of the Equation

$$2^{\cos(2x)} + 1 = 3 * 2^{-\sin^2 x} \text{ in interval } [0, \pi]$$

Find value of $\cos(6^\circ)\cos(42^\circ)\cos(66^\circ)\cos(78^\circ)$

$\underline{\underline{\text{Sol}^n}}$ Let say $E = \cos(6)\cos(42)\cos(66)\cos(78)$

$$= \frac{1}{4}\{2\cos(66)\cos(6)\} * \{2\cos(78)\cos(42)\}$$

Using $2\cos(A)\cos(B) = \cos(A+B) + \cos(A-B)$

$$E = \frac{1}{4}\{\cos(66+6)+\cos(66-6)\} * \{\cos(78+42)+\cos(78-42)\}$$

$$= \frac{1}{4}\{\cos 72 + \cos 60\} * \{\cos(120) + \cos(36)\}$$

$$= \frac{1}{4}\left\{\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right\} * \left\{-\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right\}$$

$$= \frac{1}{4}\left\{\frac{\sqrt{5}+1}{\ }\right\}\left\{\frac{\sqrt{5}-1}{\ }\right\} = \frac{1}{\ }$$ Ans

Find domain of $g(x) = \dfrac{e^x + 7}{\sqrt{x^2 - 5|x| + 6}}$

Sol^n :- For $g(x)$ to be defined we must have

$x^2 - 5|x| + 6 > 0$

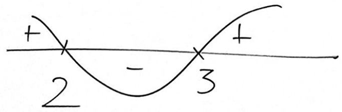
$\Rightarrow |x|^2 - 5|x| + 6 > 0$ becoz $x^2 = |x|^2$

Let say $|x| = t$

$\Rightarrow t^2 - 5t + 6 > 0$

$\Rightarrow (t-2)(t-3) > 0$

$\Rightarrow$



So $t < 2$ or $t > 3$

$\Rightarrow |x| < 2$ or $|x| > 3$

$\Rightarrow x \in (-2, 2)$ or $x \in (-\infty, -3) \cup (3, +\infty)$