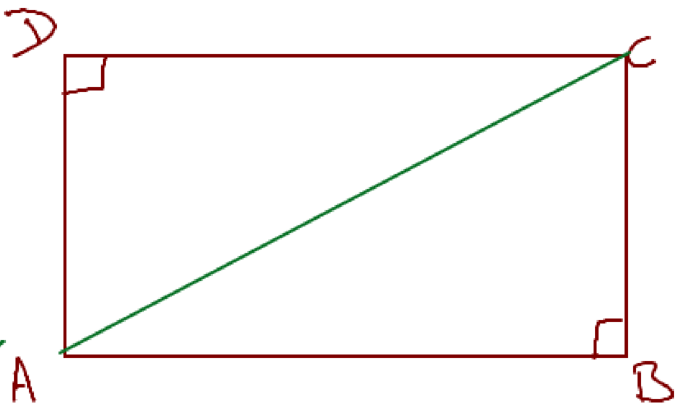
8.

To Prove:

$$2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$$



given $\angle B = \angle D = 90^\circ$ ..

In ΔABC by Pythagoras Th

$$AC^2 = \underline{AB^2 + BC^2} \quad \text{---- (i)}$$

Similarly in ΔADC by Pythagoras Theorem

$$AC^2 = AD^2 + DC^2 \quad \text{---- (ii)}$$

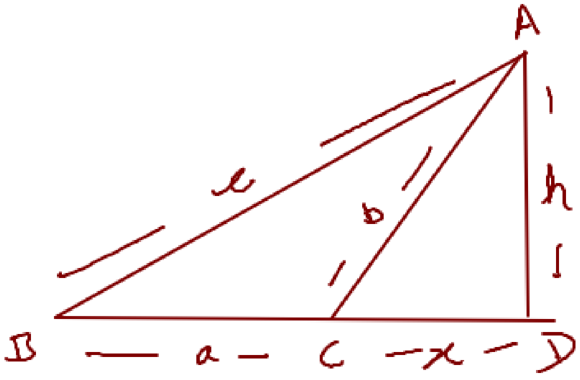
(i) + (ii)

$$AC^2 + AC^2 = AB^2 + BC^2 + AD^2 + DC^2$$

$$2AC^2 - BC^2 = AB^2 + AD^2 + DC^2$$

Proved

9. $c^2 = a^2 + b^2 + 2ax$



In Δ ABD

$$AB^2 = AD^2 + BD^2$$

$$c^2 = h^2 + (a+x)^2$$

$$c^2 = h^2 + a^2 + x^2 + 2ax \quad \text{- - - - - - (i)}$$

In Δ ACD

$$AC^2 = AD^2 + DC^2$$

$$b^2 = h^2 + x^2 \quad \text{- - - - - - (ii)}$$

(i) − (ii)

$$c^2 - b^2 = (h^2 + a^2 + x^2 + 2ax) - (h^2 + x^2)$$

$$c^2 - b^2 = \cancel{h^2} + a^2 + \cancel{x^2} + 2ax - \cancel{h^2} - \cancel{x^2}$$

$$c^2 - b^2 = a^2 + 2ax \Rightarrow c^2 = a^2 + b^2 + 2ax$$

Proved

10. given $AB = 5\,AE$ & $AC = EI$

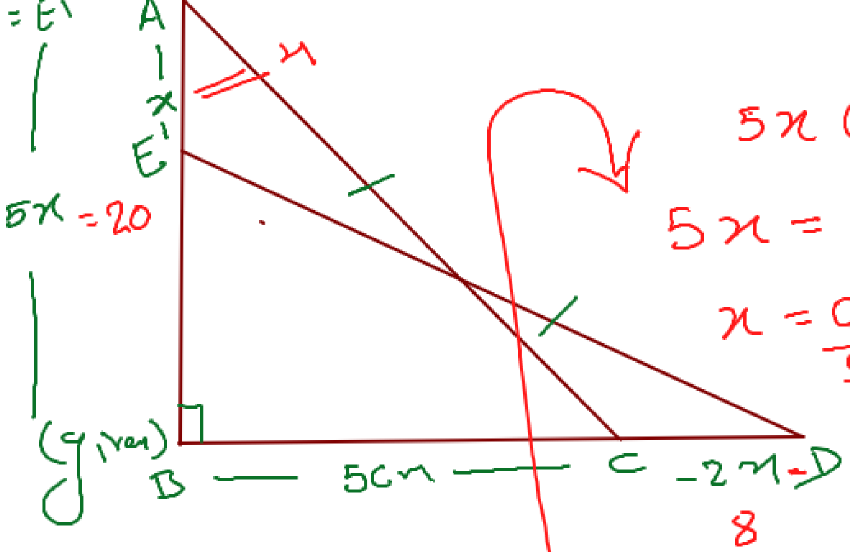
Let $\underline{AE = x}$

$AB = 5\,AE$

$AB = 5x$

Similarly, $CD = 2\,AE$

$CD = 2x$



$5x(x-4) = 0$

$5x = 0$ or $x - 4 = 0$

$x = \frac{0}{5} = 0$ or $x = 4$

$x = 0$ is not possible

Hence : $x = 4$

In Δ ABC

$AC^2 = AB^2 + BC^2$

$AC^2 = (5x)^2 + (5)^2$

$AC^2 = 25x^2 + 25 \quad \ldots (i)$

$AC^2 =$

In Δ EBD

$ED^2 = EB^2 + BD^2$

$ED^2 = (4x)^2 + (5+2x)^2$

$ED^2 = 16x^2 + 25 + 4x^2 + 20x$

$ED^2 = 20x^2 + 20x + 25 \quad \ldots (ii)$

$(\because AC = ED \Rightarrow AC^2 = ED^2)$

$AC^2 = 25 \times 16 + 25$

$= 400 + 25$

$AC^2 = 425$

$AC = 5\sqrt{17}$

From eqn (i) & (ii)

$25x^2 + 25 = 20x^2 + 20x + 25$

$25x^2 - 20x^2 - 20x = -25 + 25$

$5x^2 - 20x = 0 \Rightarrow 5x(x-4) = 0$

5	425
5	85
17	17

$$\sqrt{5 \times 5 \times 17}$$

$$5\sqrt{17}$$