Practice Set: Applications of Derivatives

Mean Value Theorems

May 12, 2024

Class XII — Time: 30 min

Subjective Type

1. Suppose that $f(0) = -3$ and $f'(x) \leq 5,\ \forall\ x$. Then, what is the largest value which $f(2)$ can attain?

2. Prove that for $\beta > 1$,the equation $x \log x + x = \beta$ has at least one solution in $[1, \beta]$.

3. If $f(x)$ is continuous and differentiable in $[-3, 9]$ and $f'(x) \in [-2, 8],\ \forall\, x \in (-3, 9)$. Let N be the number of divisors of the greatest possible value of $f(9) - f(-3)$, then find the sum of digits of N.

Single Choice Type

4. Let the function

$$f(x) = \begin{cases} \cos^{-1} x, & -1 \leq x \leq 0 \\ mx + c, & 0 < x \leq 1 \end{cases}$$

satisfies Lagrange's mean value theorem in $[-1, 1]$ then ordered pair (m, c) is :

(a) $\left(1, -\frac{\pi}{2}\right)$ (b) $\left(1, \frac{\pi}{2}\right)$ (c) $\left(-1, -\frac{\pi}{2}\right)$ (d) $\left(-1, \frac{\pi}{2}\right)$

5. If $c = \frac{1}{2}$ and $f(x) = 2x - x^2$, then the interval of x on which LMVT is applicable is

(a) (1,2) (b) (-1,1) (c) (0,1) (d) (2,1)

6. Given$f'(1) = 1$ and $f(2x) = f(x), \forall\, x > 0$.If $f'(x)$ is differentiable then there exists a number $c \in (2, 4)$ such that $f''(c)$ equals

(a) $\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) $-\frac{1}{8}$

Comprehension Type

Let $f(x)$ be a function such that its derivative $f'(x)$ is continuous in $[a, b]$ and differentiable in (a, b). Consider a function $\phi(x)$

$$= f(b) - f(x) - (b - x)f'(x) - (b - x)^2 A.$$

If Rolle's theorem is applicable to $\phi(x)$ on $[a, b]$, answer following questions.

7. If there exists some number $c(a < c < b)$ such that $\phi'(c) = 0$ and

$$f(b) = f(a) + (b - a)f'(a) + \lambda(b - a)^2 f''(c),$$

then λ is

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

8. Let $f(x) = \sin x, a = \alpha$ and $b = \alpha + h$. If there exists real number t such that $0 < t < 1$, $\phi'(\alpha + th) = 0$ and

$$\frac{\sin(\alpha + h) - \sin \alpha - h \cos \alpha}{h^2} = \lambda \sin(\alpha + th),$$

then $\lambda =$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

9. Let $f(x) = x^3 - 3x + 3$, $a = 1$ and $b = 1 + h$. If there exists $c \in (1,\ 1 + h)$ such that $\phi'(c) = 0$ and $\dfrac{f(1 + h) - f(1)}{h^2} = \lambda c$, then $\lambda =$

(a) 1/2 (b) 2 (c) 3 (d) does not exist